AIM-CEA Saclay, France

Signal and Image Processing in Astrophysics

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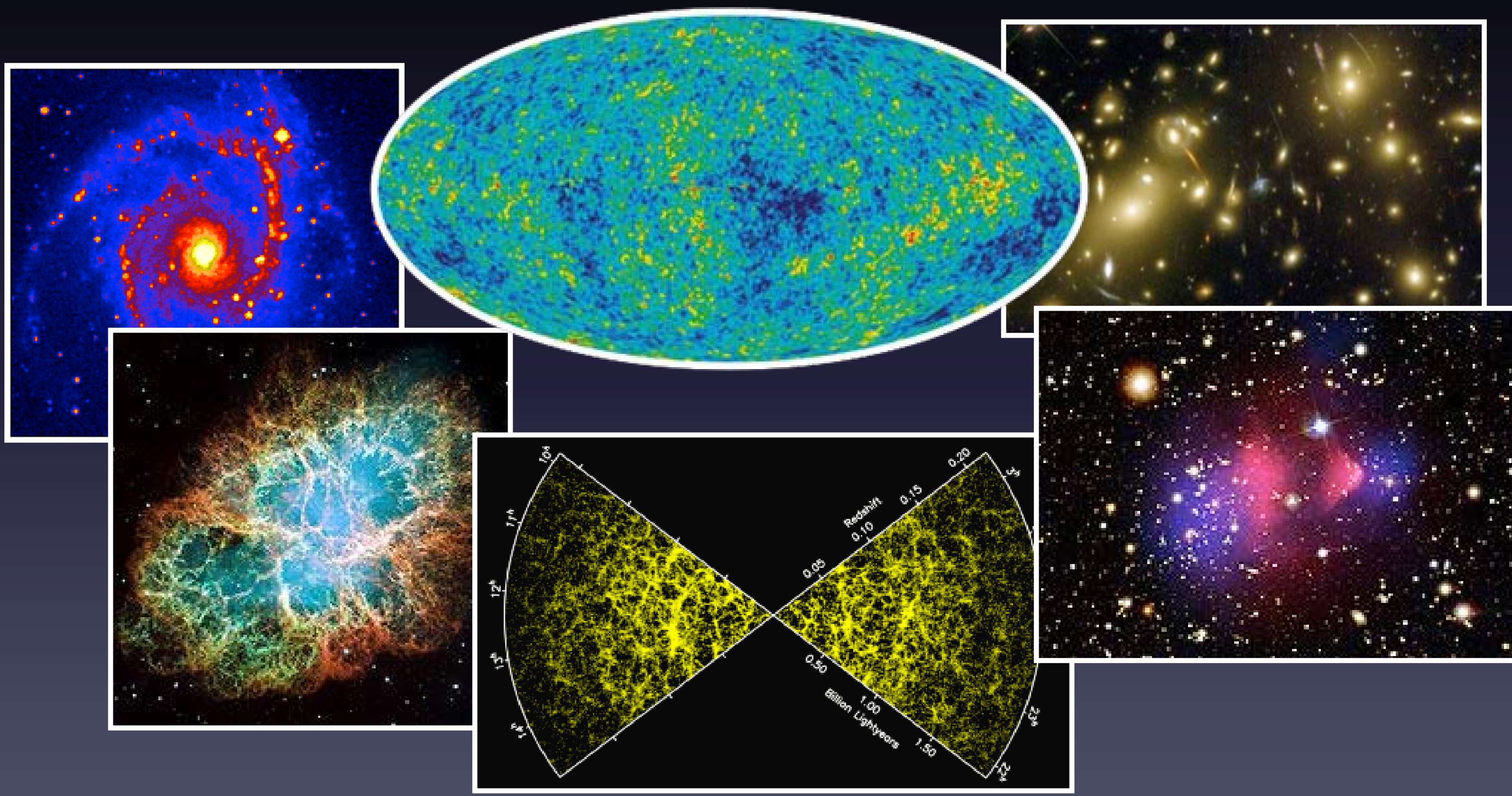


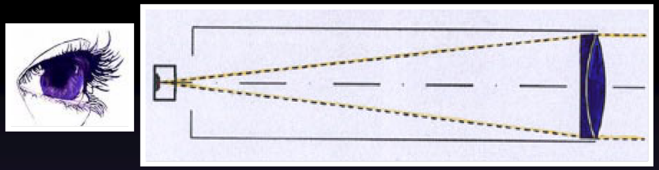
Image Processing : Goals

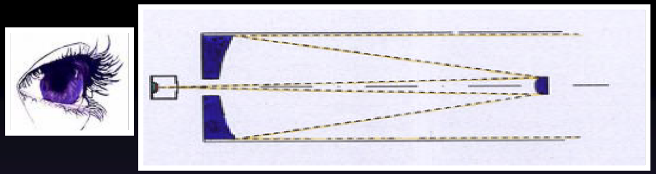
Image processing is used once the image acquisition is done by the telescope

- ✓ Correct from the problems encountered during acquisition:
  - ✓ Reduce the instrumental and atmospheric effects
  - ✓ Reduce the observation noise
  - ✓ Deal with missing data (partial sky coverage, defective pixel…)
- ✓ Extract the useful information to enable physical interpretation
  - ✓Compressed sensing
  - ✓Source separation

=> Impact the instrument design

How to reduce atmospheric and instrumental effects ?

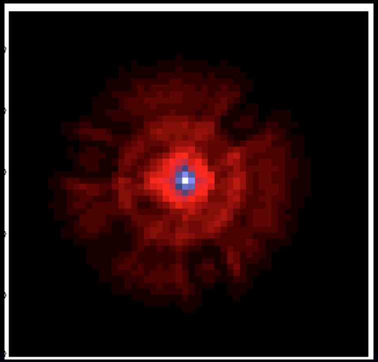




Hale Telescope (5 m) at Mont Palomar observatory (alt. 1706 m), California

Great Refractor (76 cm) at Nice observatory

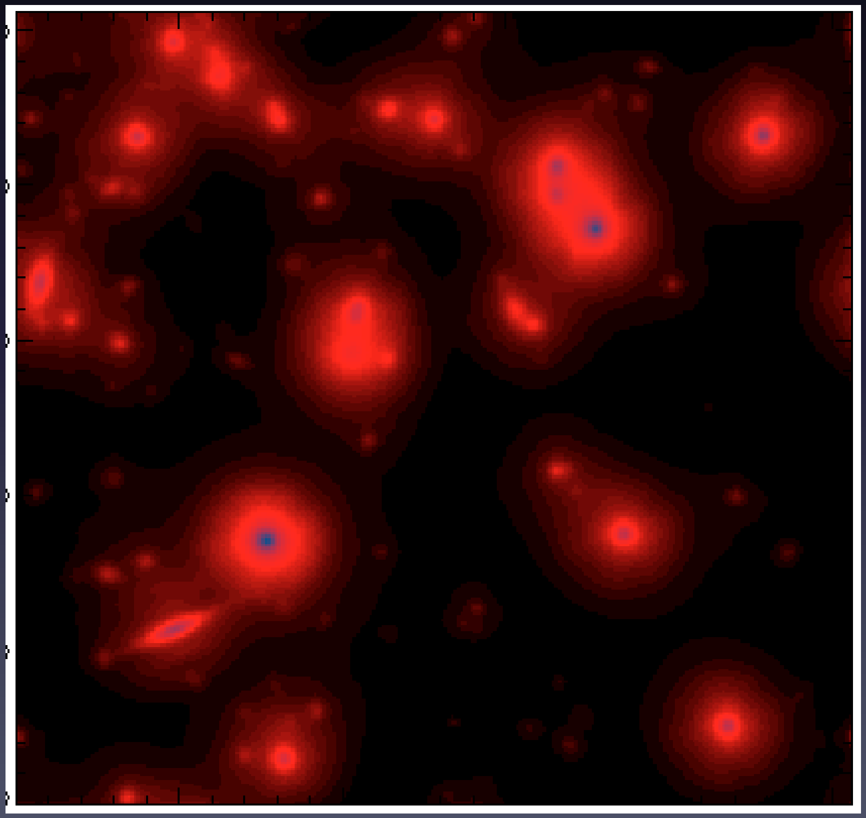
Hubble Space Telescope (2.4m)



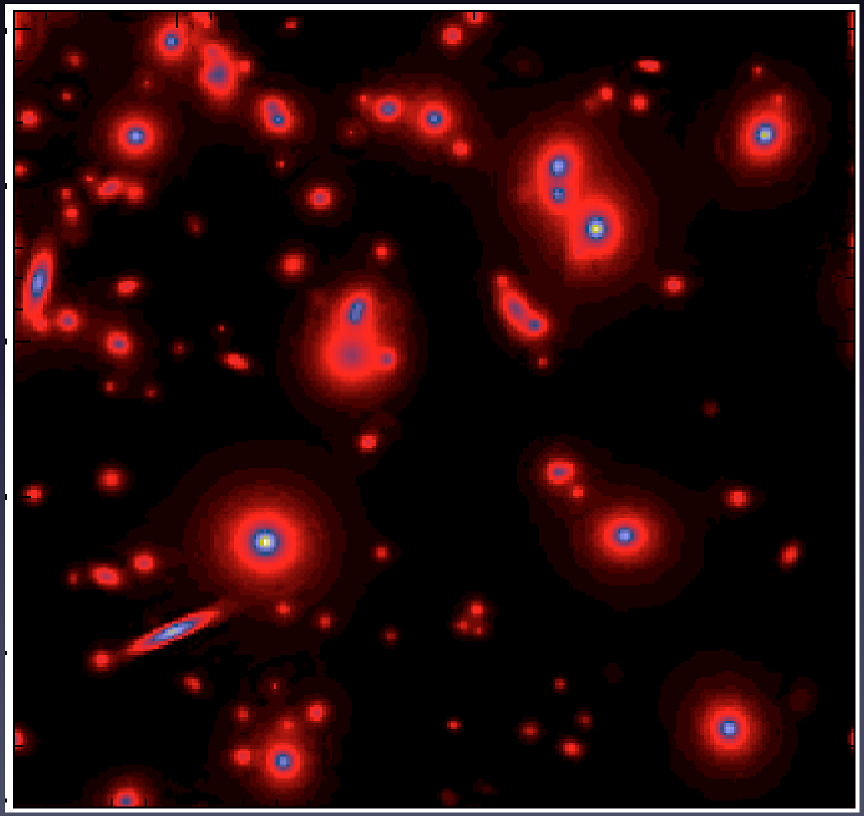
PSF

PSF correction

$$I = O * H$$

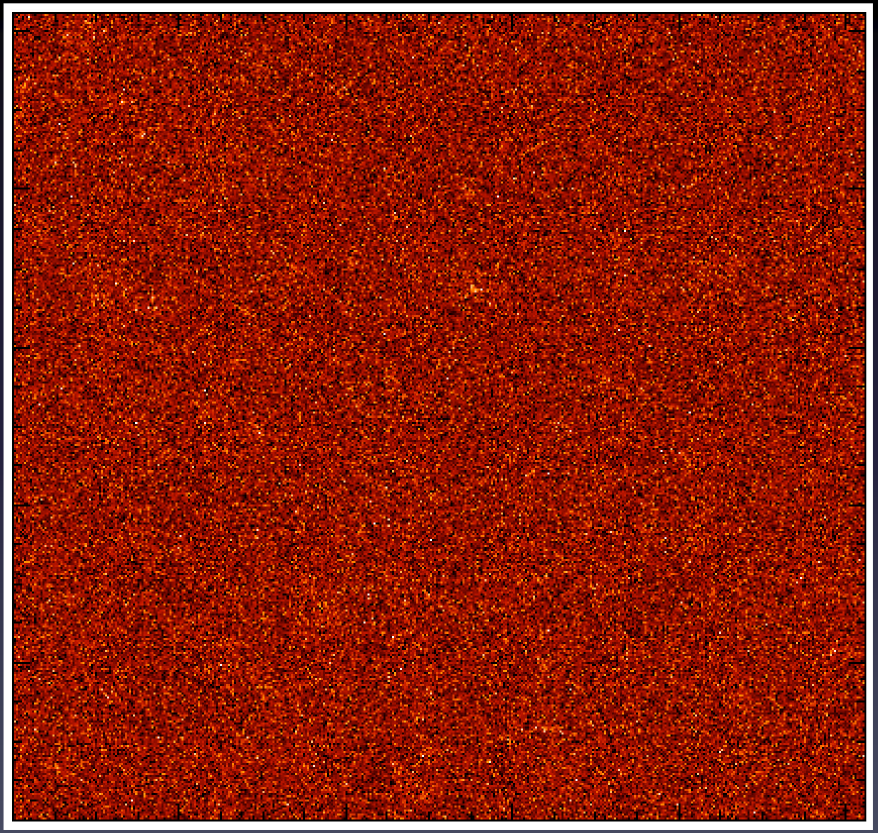


Observed image



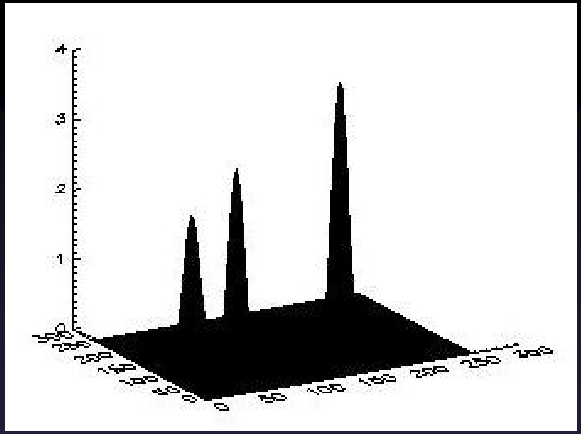
Convolved image

How to reduce the observational noise ?

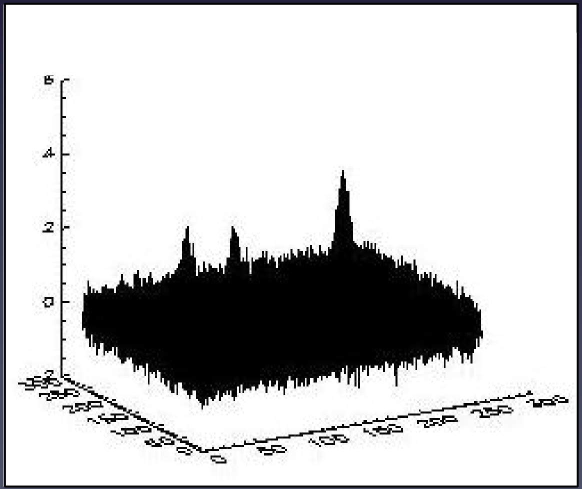


Standard methods based on a linear filter

(i.e. Gaussian filtering)

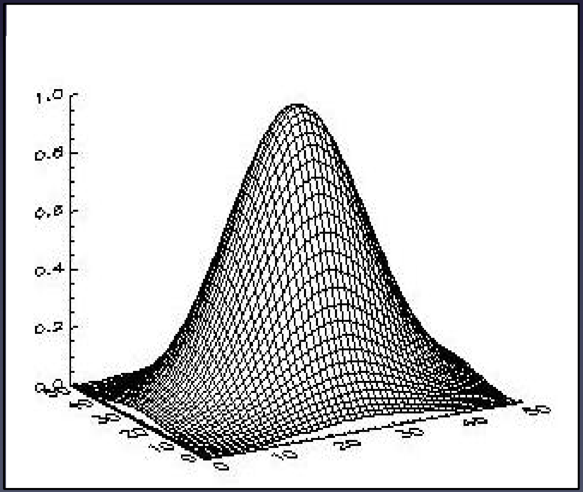


Signal



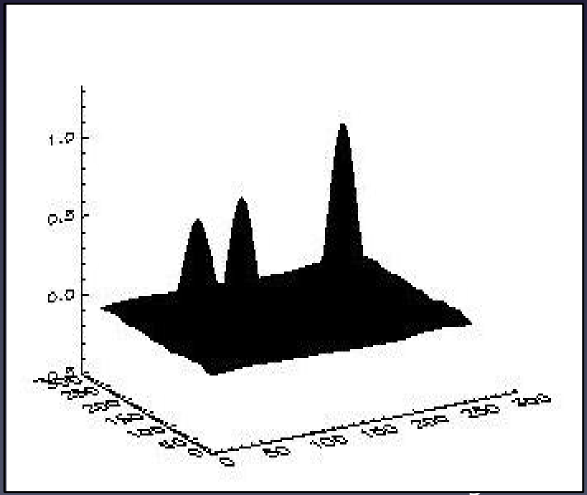
Signal + noise

⊗



Gaussian function (σ)

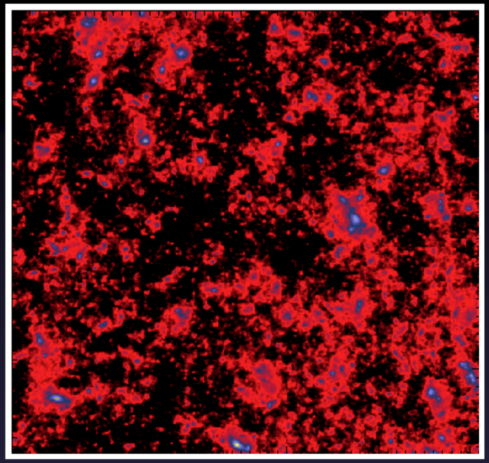
=



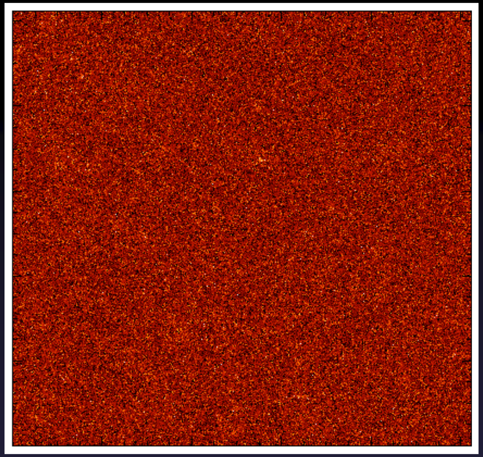
Filtered signal

Standard methods based on a linear filter

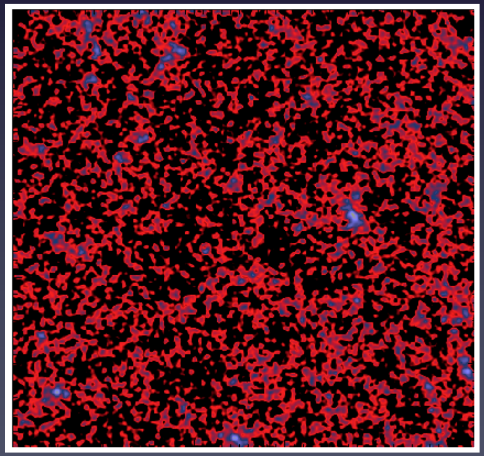
(i.e. Gaussian filtering)



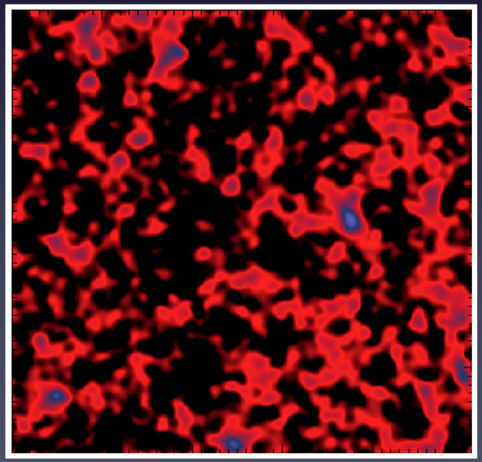
Signal



Signal + noise



Gaussian filtered (σ =2 pixels)

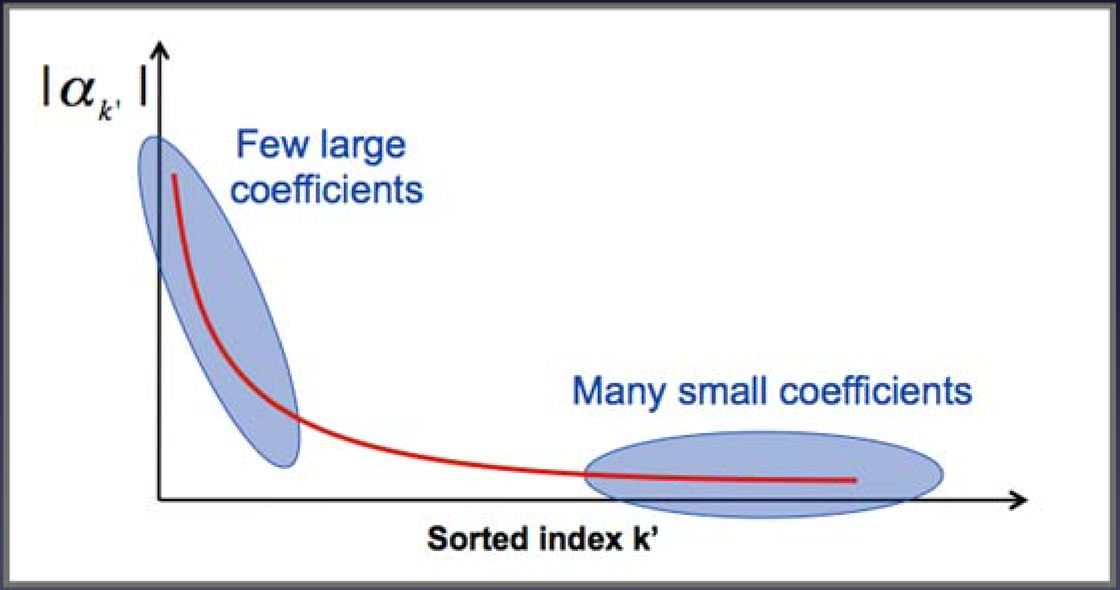


Gaussian filtered (σ =5 pixels)

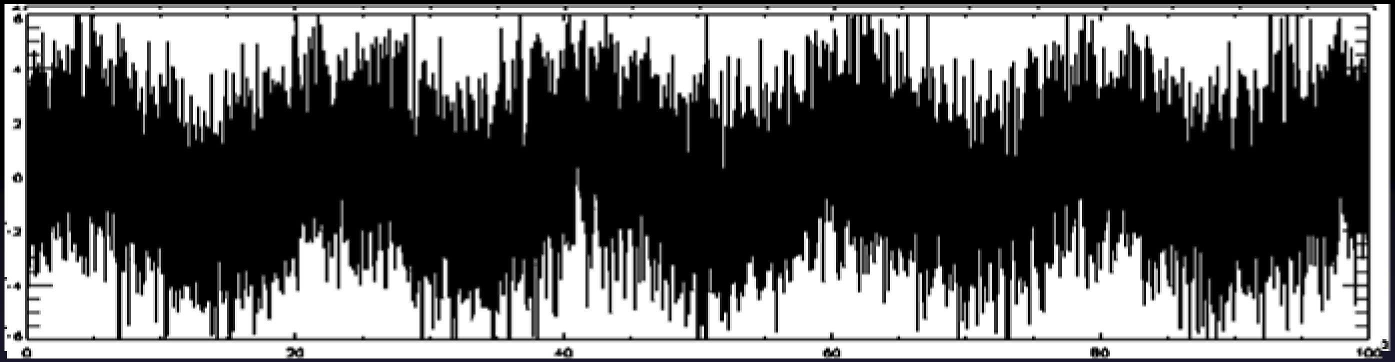
Methods based on sparsity

Considering a transform : $\alpha = \Phi^T X$

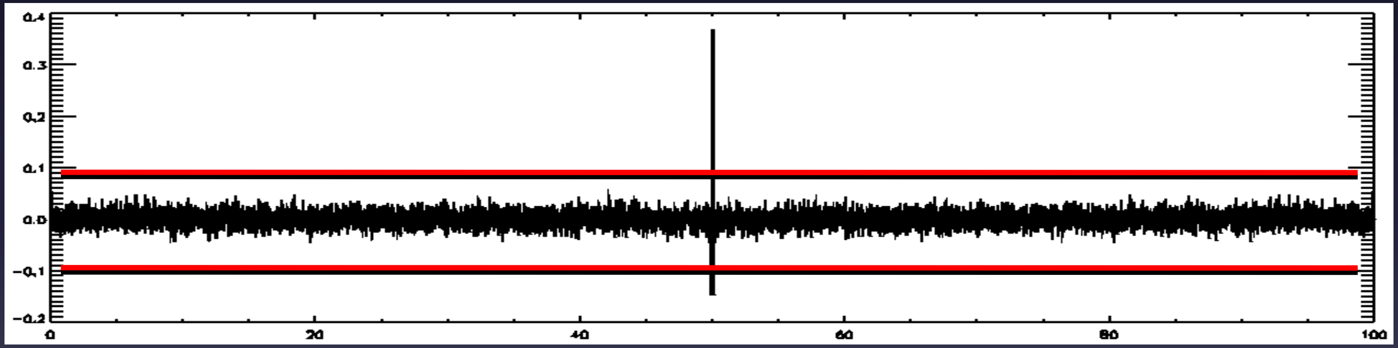
A signal X is sparse in a basis Φ
if most of the coefficients α are equal to zero or close to zero



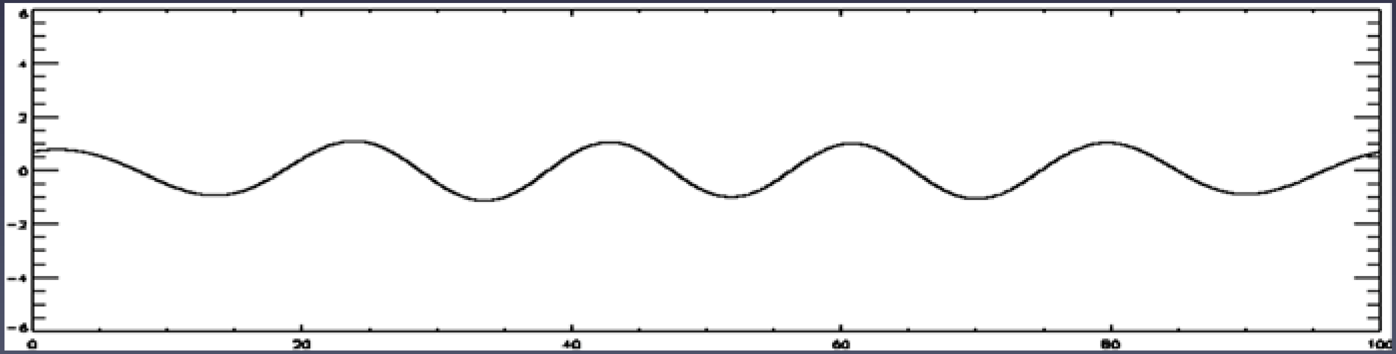
Basic Example



Original signal / *Noisy signal*



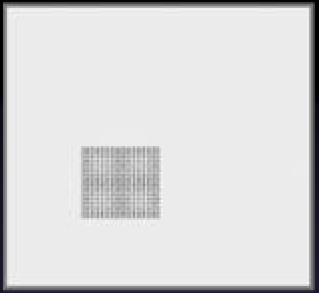
Fourier Transform



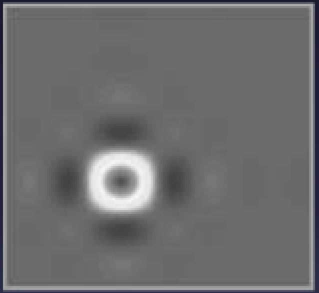
Filtered Signal

Signal and image representations

- ✓ Local DCT :
  - ✓ Stationary textures
  - ✓ Locally oscillatory

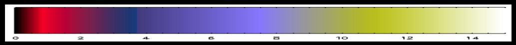


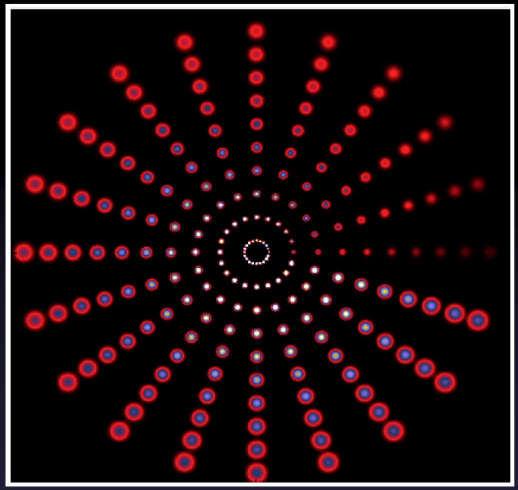
- ✓ Wavelet Transform
  - ✓ Piecewise smooth
  - ✓ Isotropic structures



- ✓ Curvelet Transform
  - ✓ Piecewise smooth
  - ✓ Edge structures

Adapted Representations





Test image 1

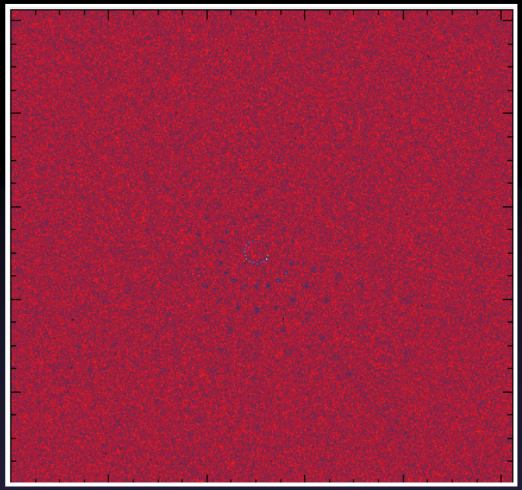
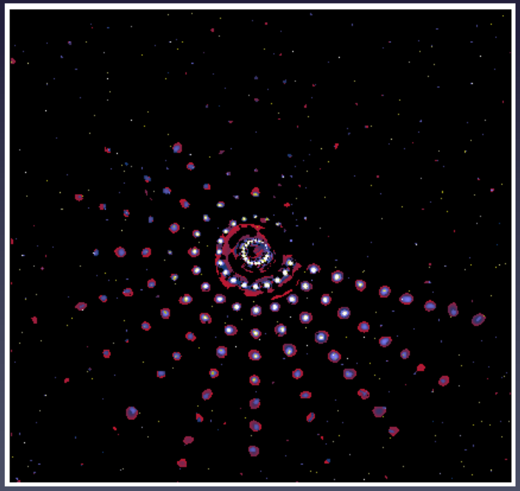
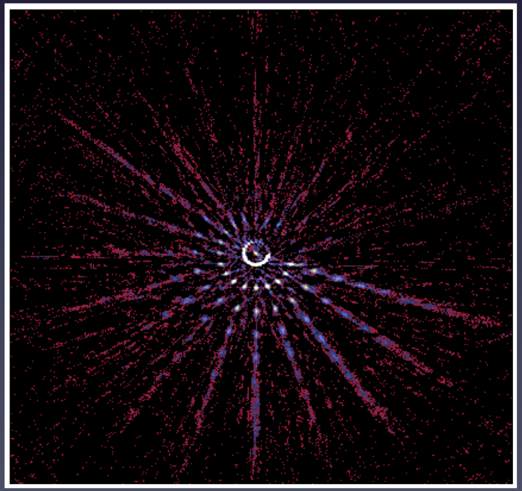


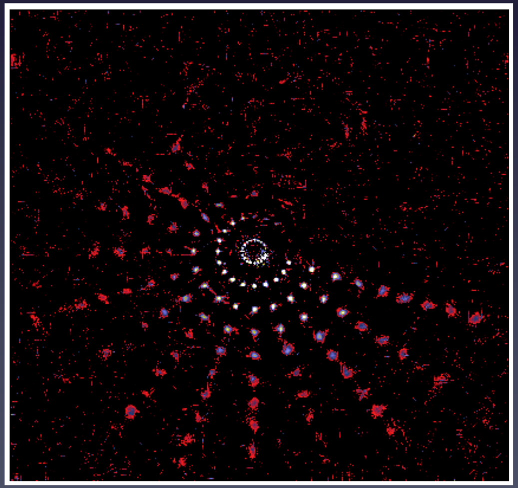
Image test 1 + noise



Wavelet filtering



Ridgelet filtering



Curvelet filtering

Adapted Representations

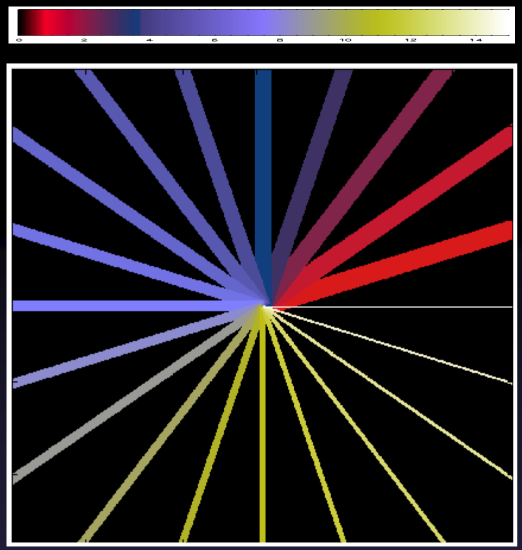


Image test 2

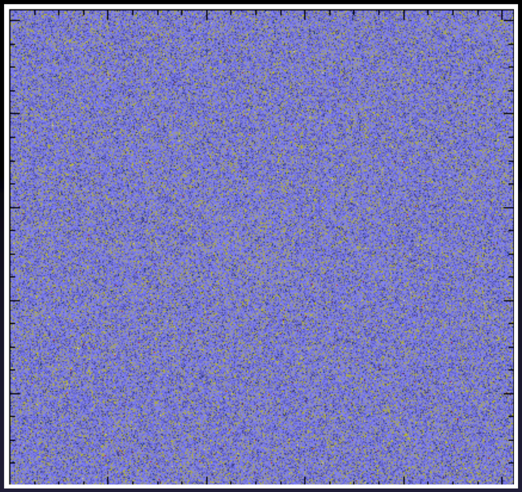
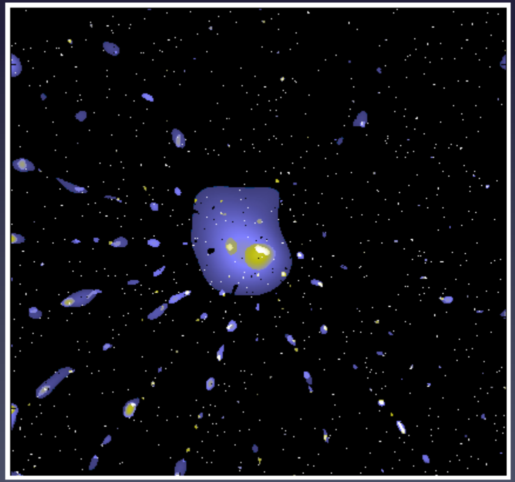
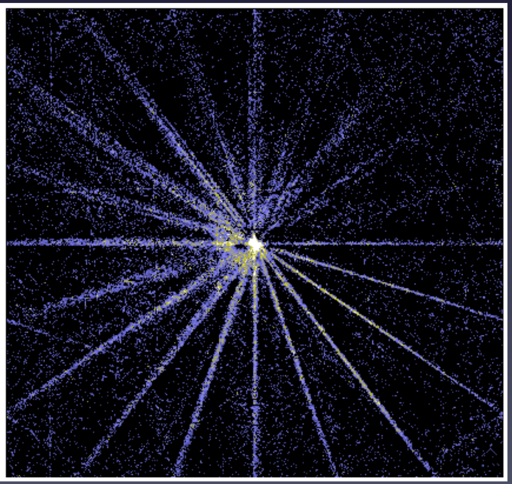


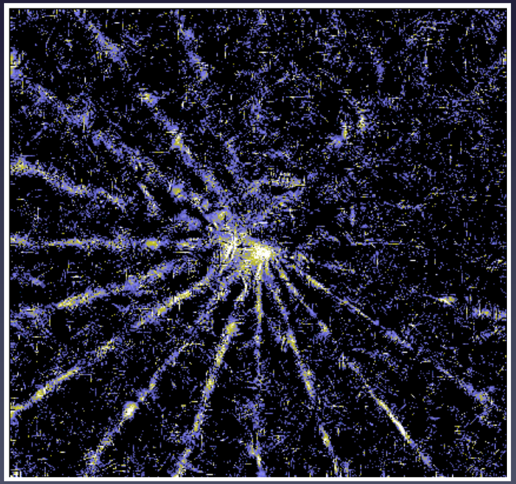
Image test 2 + noise



Wavelet filtering

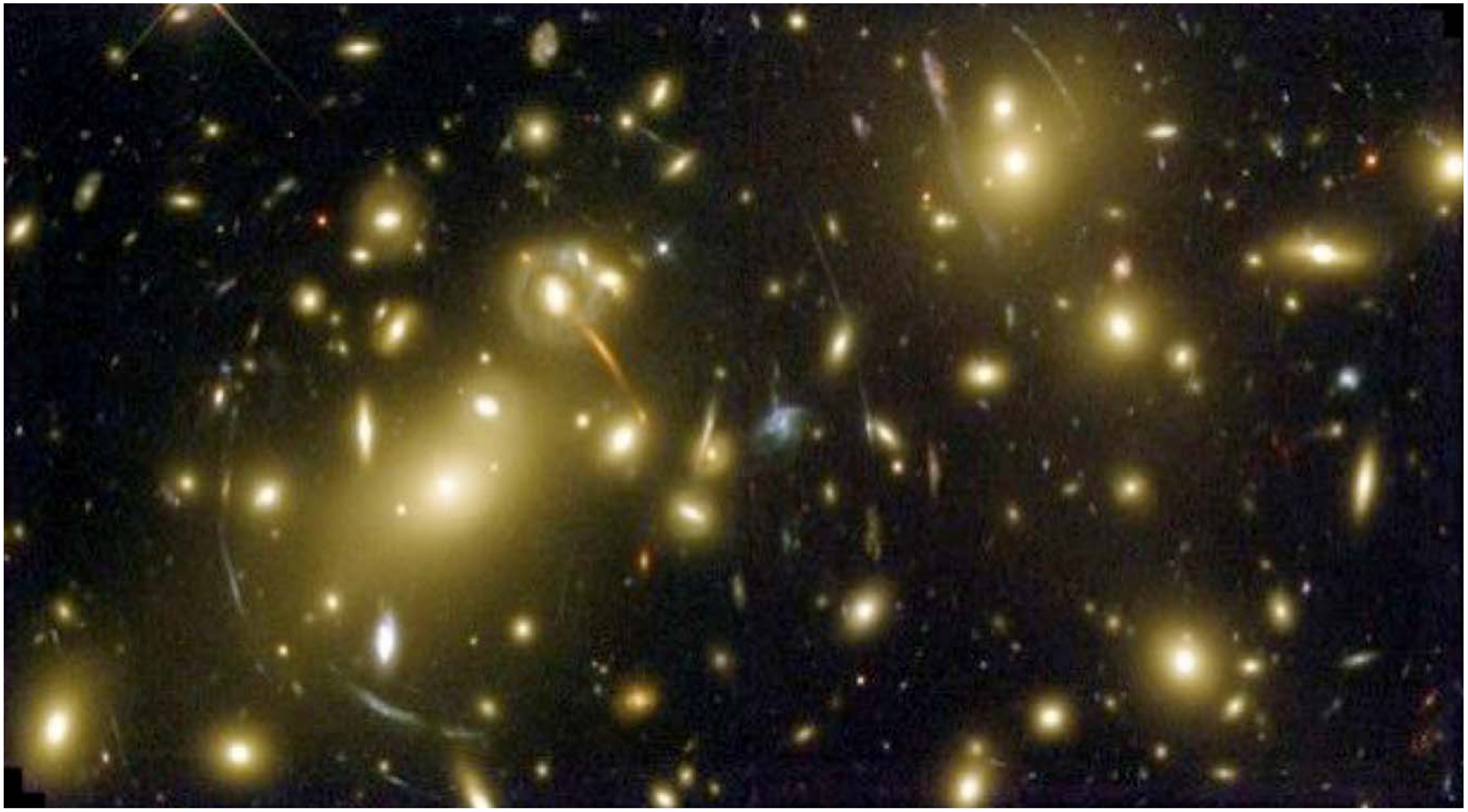


Ridgelet filtering



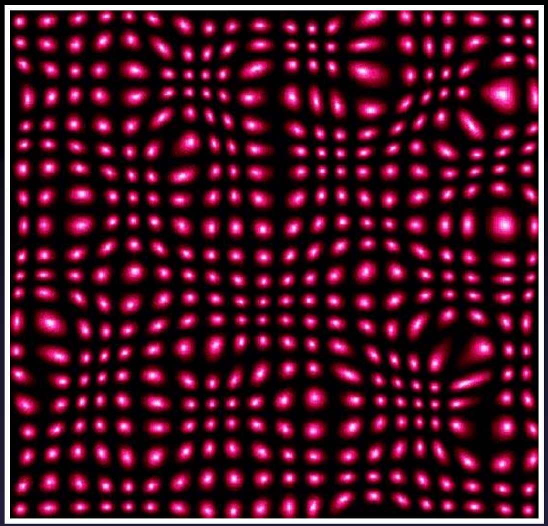
Curvelet filtering

Gravitational Lensing effect observed by the Hubble Space Telescope

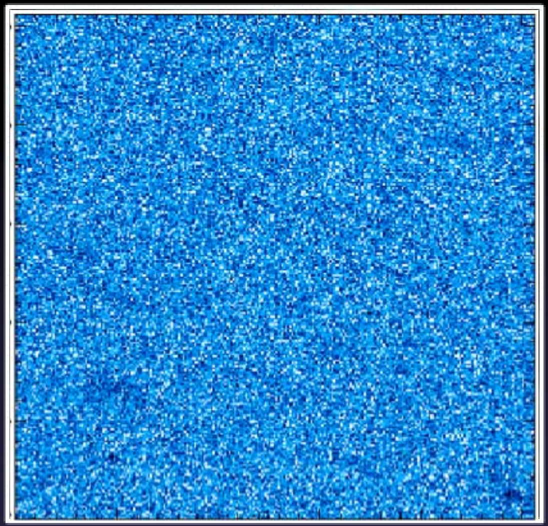


Cluster Abell 2218

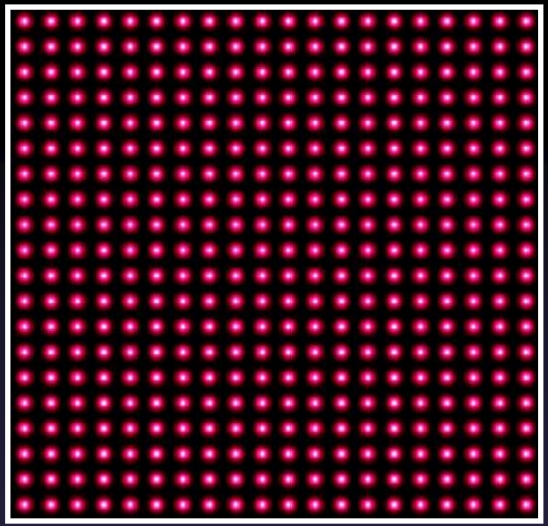
Weak Gravitational Lensing



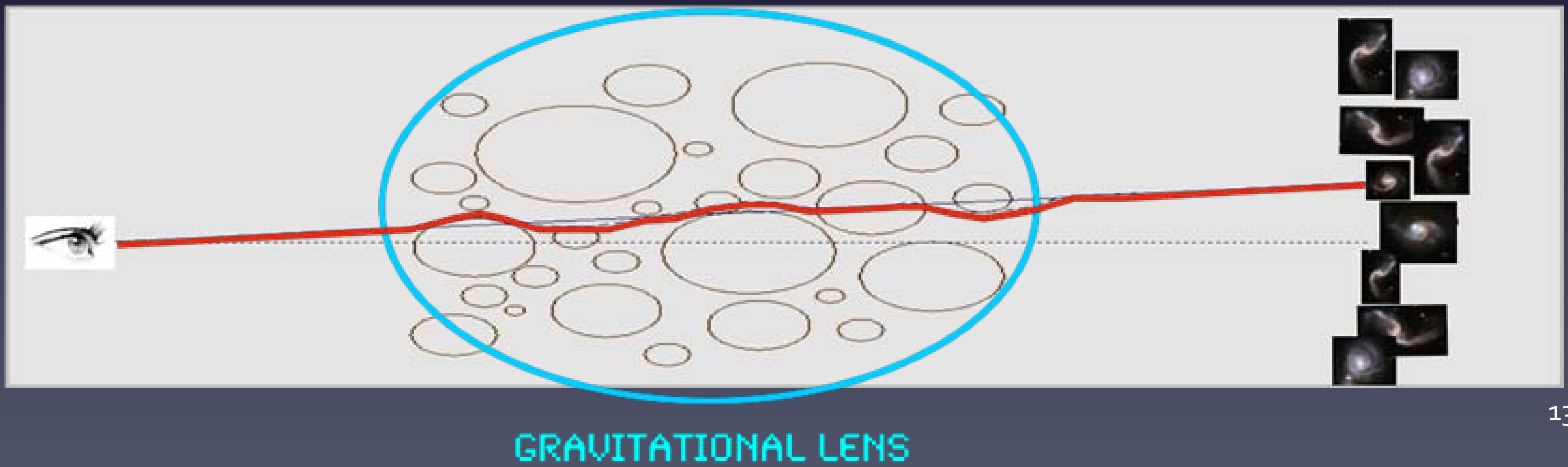
Observer



Gravitational lens

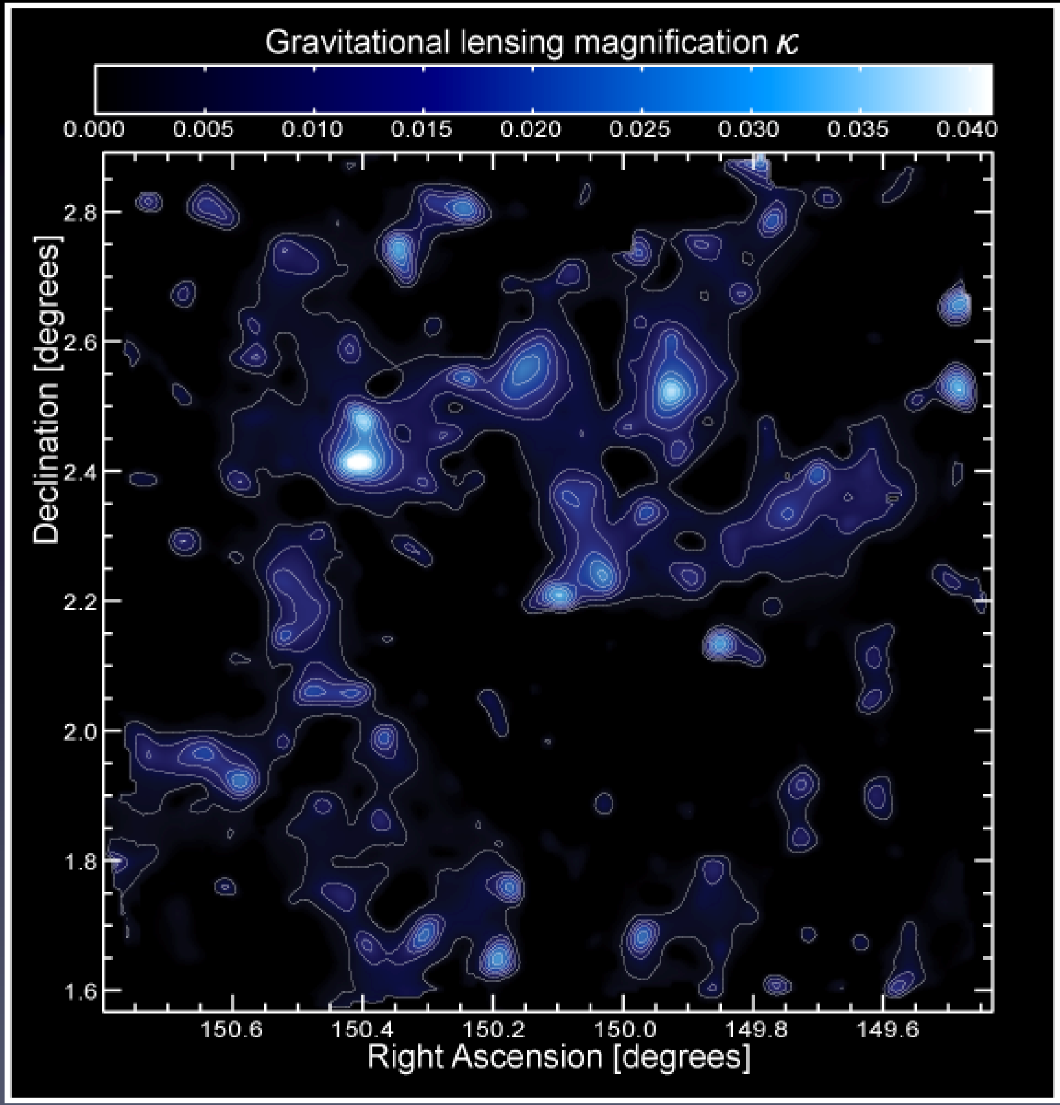


Background galaxies



Dark matter Map

- HST observations -



Missing data

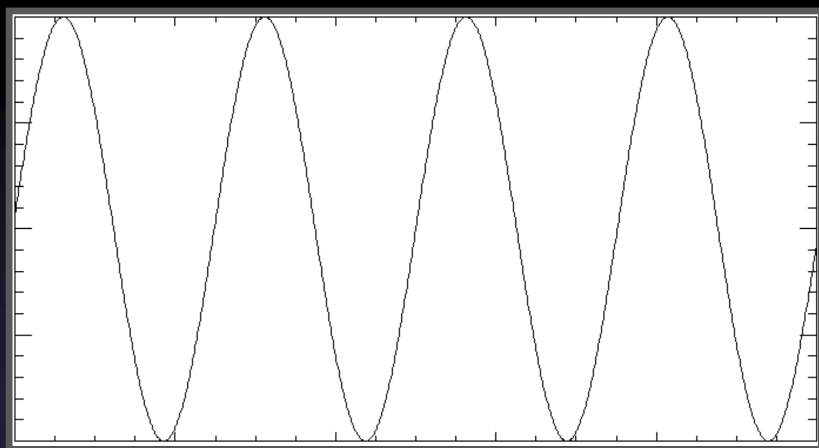
- ✓ Causes of missing data:
  - ✓ Occurrence of defective or dead pixels
  - ✓ Partial sky coverage due to problems in the scan strategy
  - ✓ Saturated pixels
  - ✓ Absorption or masking of the signal by a foreground

- ✓ Problems caused by missing data:
  - ✓ Bias and decrease on statistical power
  - ✓ Distortions in the frequency domain due to abrupt truncation
  - ✓ Other edge effects in multi-scale transforms

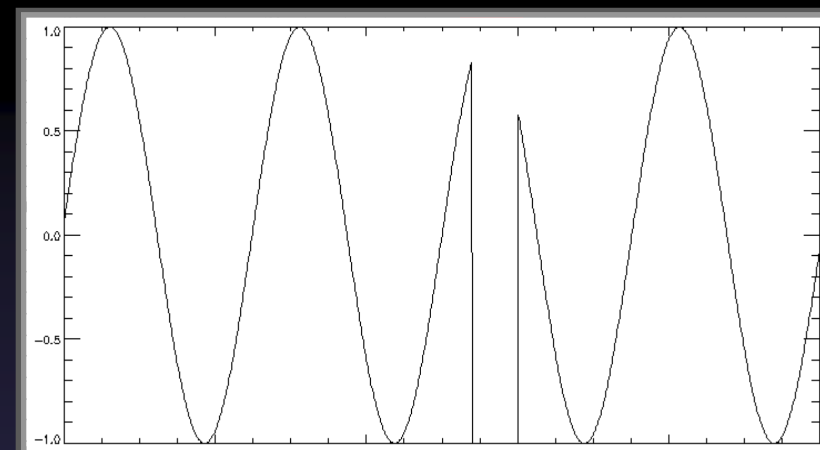
- ✓ How to deal with missing data?
  - ✓ Correction of the measure by the proportion of missing data
  - ✓ Other corrections specific to a given measure (i.e. MASTER for power spectrum estimation)
  - ✓ Inpainting methods

Inpainting based on sparsity

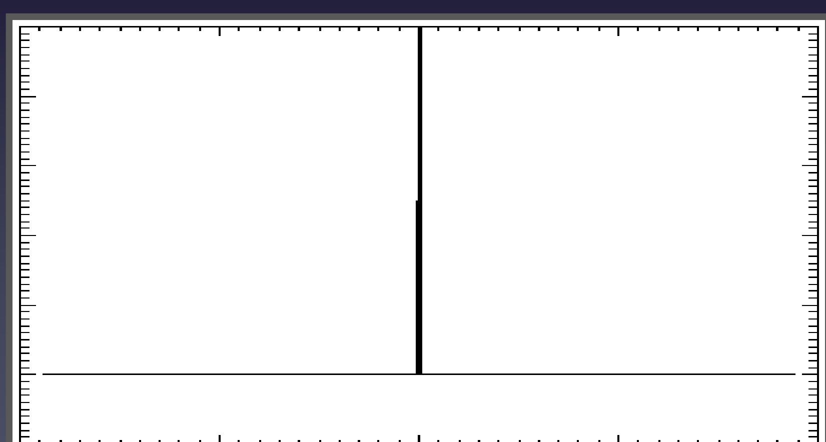
$$\min_{\alpha} ||\alpha||_1 \text{ s.t. } y = M\Phi\alpha$$



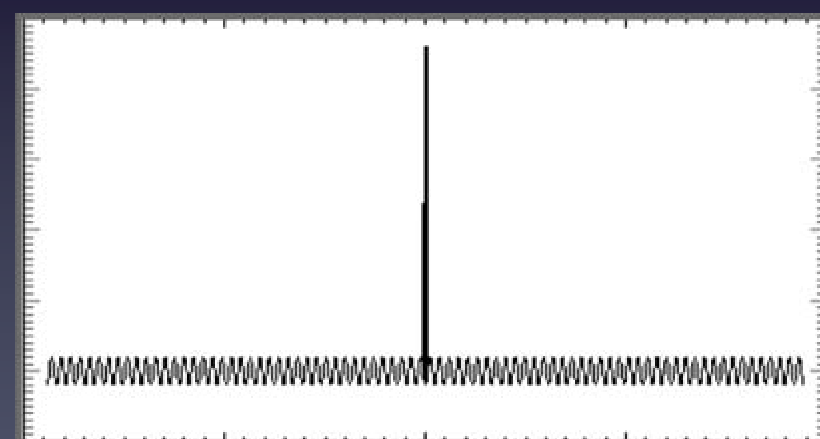
Sine curve



Truncated sine curve



Fourier transform of the sine curve

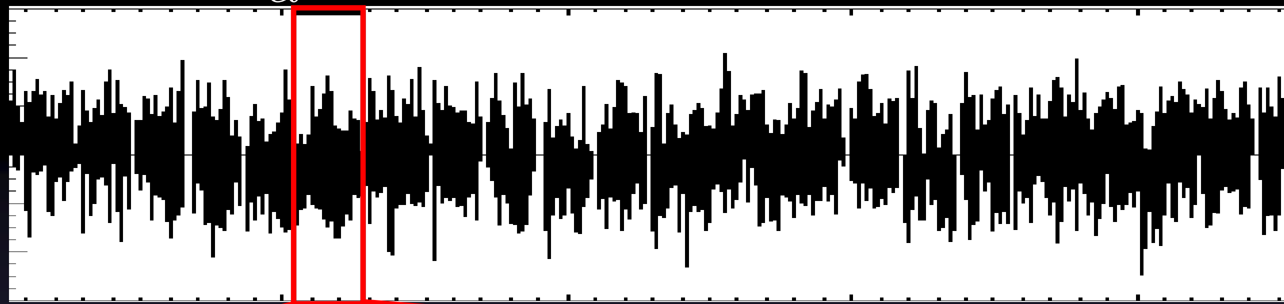


Fourier transform of the truncated sine curve

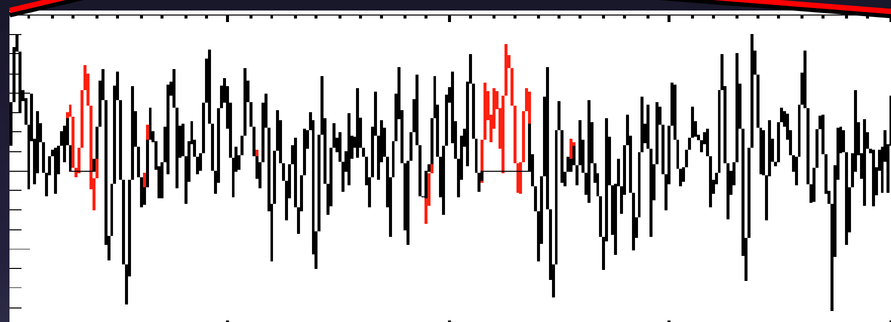
Inpainting on asterosismic data

$$\min_{\alpha} ||\alpha||_1 \text{ s.t. } y = M\Phi\alpha$$

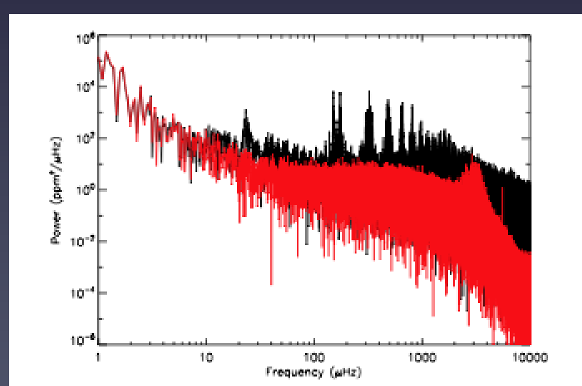
Light curve (time series)



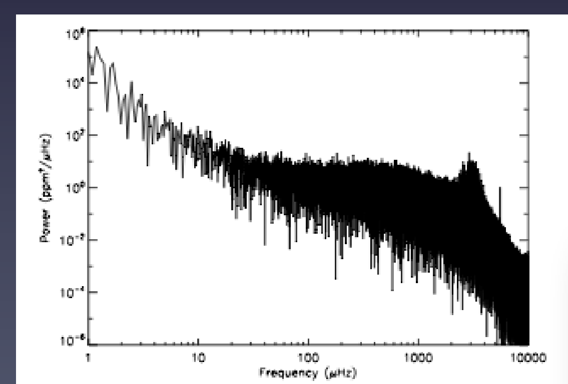
Zoom on the Light curve



Power spectrum

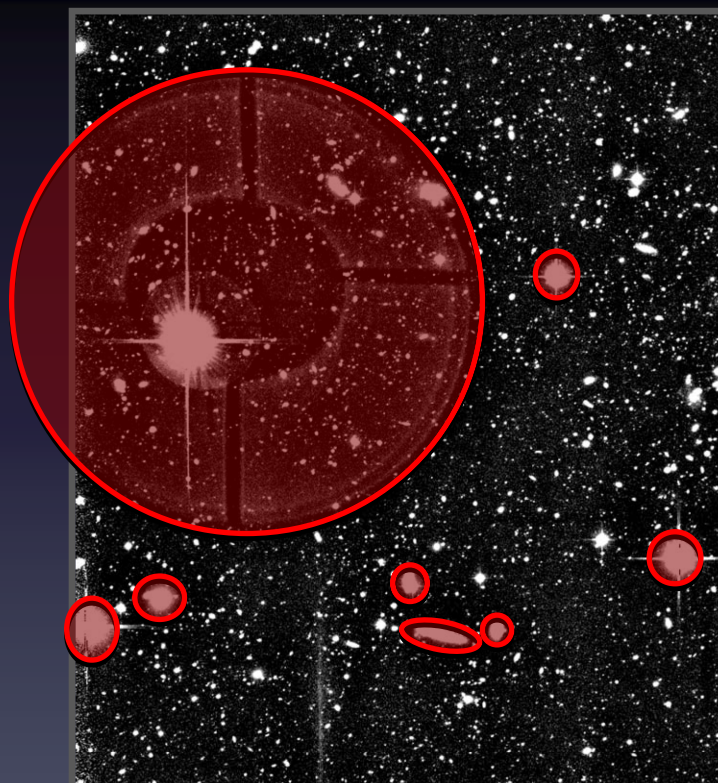


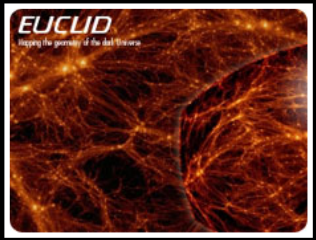
Original (red) and masked (black) data



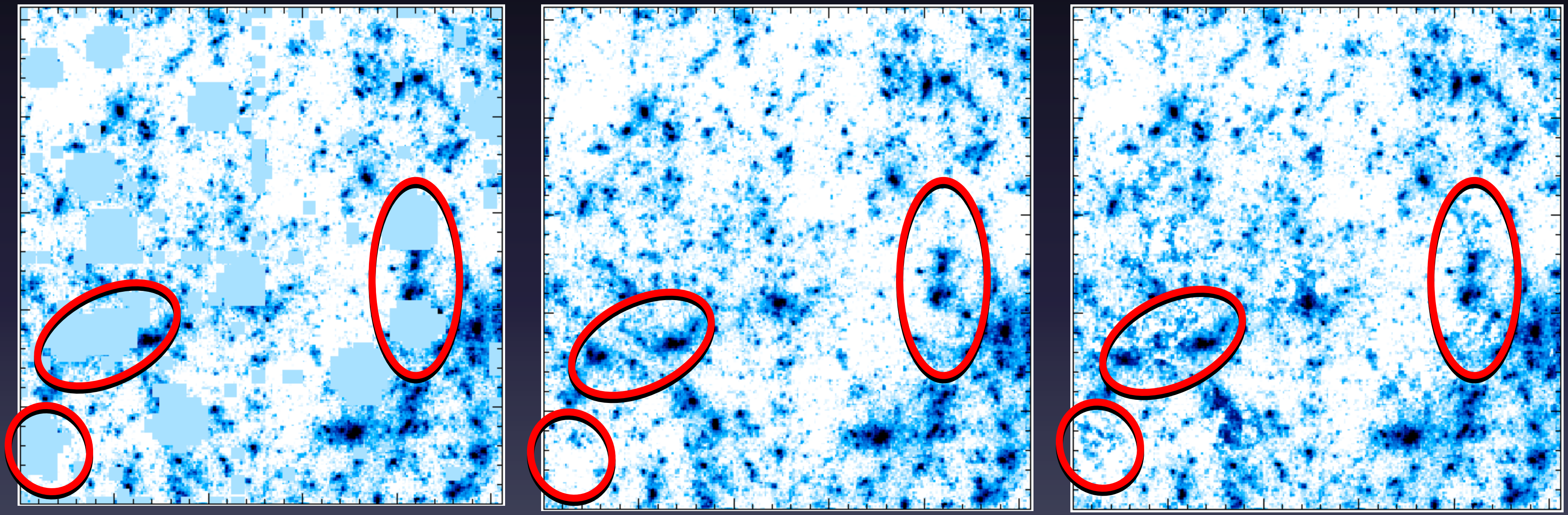
Inpainted data (black)

Missing data
In Weak Lensing

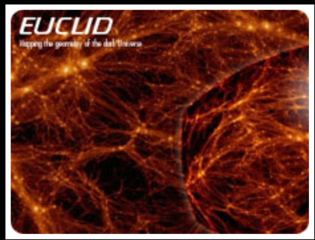




Inpainting in Weak Lensing data

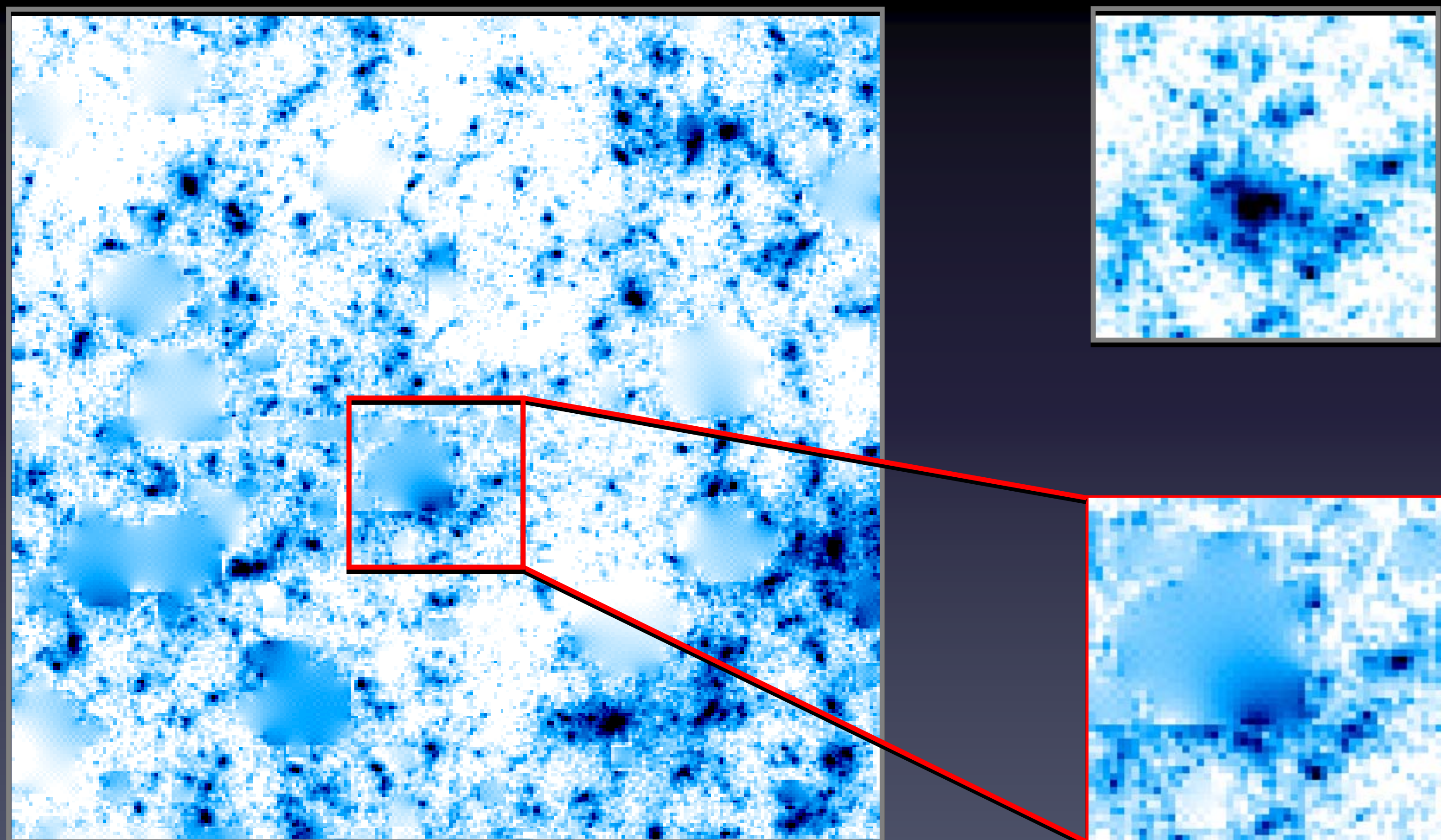


Which image is the original one ?



Inpainting in Weak Lensing data

$$\min_{\alpha} ||\alpha||_1 \text{ s.t. } y = M\Phi\alpha$$



Compressed sensing

- ✓ Shannon-Nyquist sampling theorem :
  - ✓ No loss of information if the sampling frequency is two times the highest frequency of the signal
  - ✓ The number of sensors is determined by the resolution

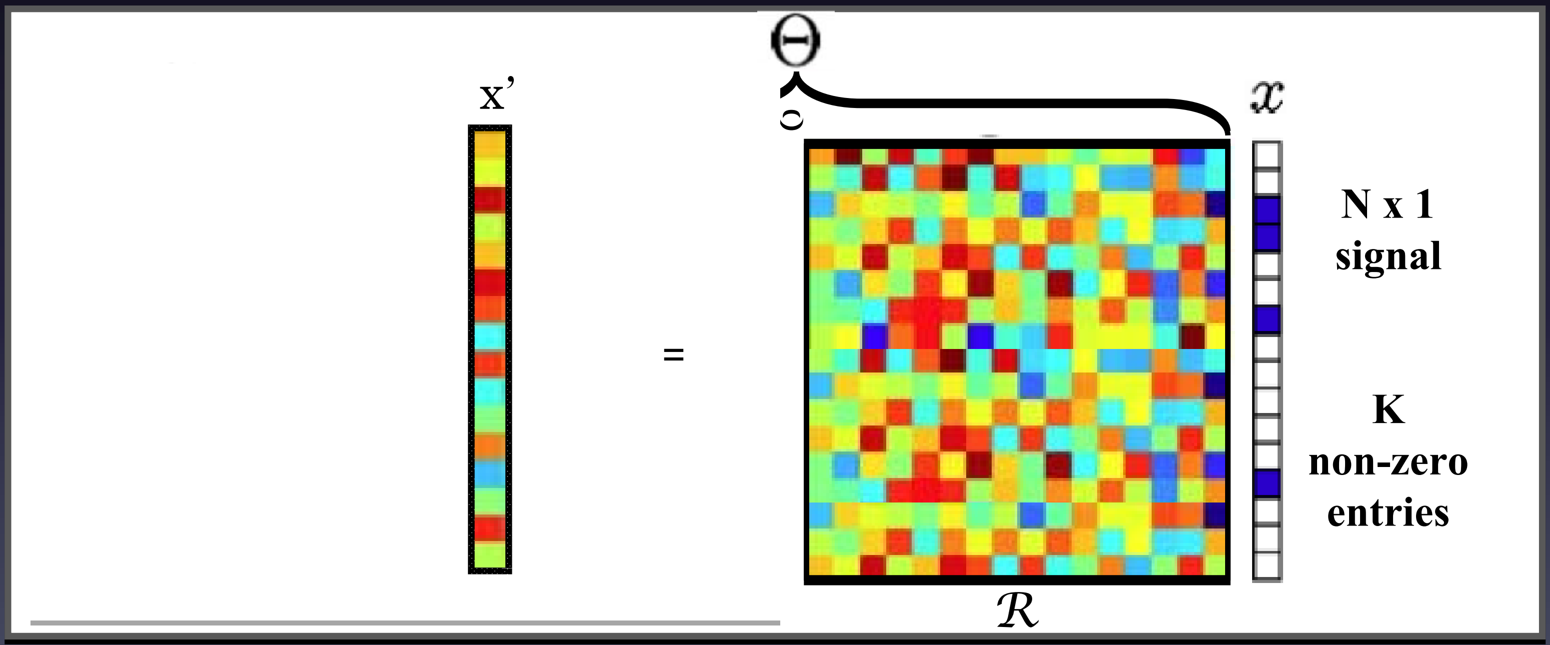
⇒ Can we get an exact recovery from a smaller number of measurements ? YES !

- ✓ Compressed sensing theorem :
  - ✓ No loss of information if :
    - the signal is local and coherent.
    - the measurements are global and decoherent.
  - ✓ The number of measurements is about K. log(N) where K is the number of non-zero entries in the signal.

Compressed sensing: A non linear sampling theorem

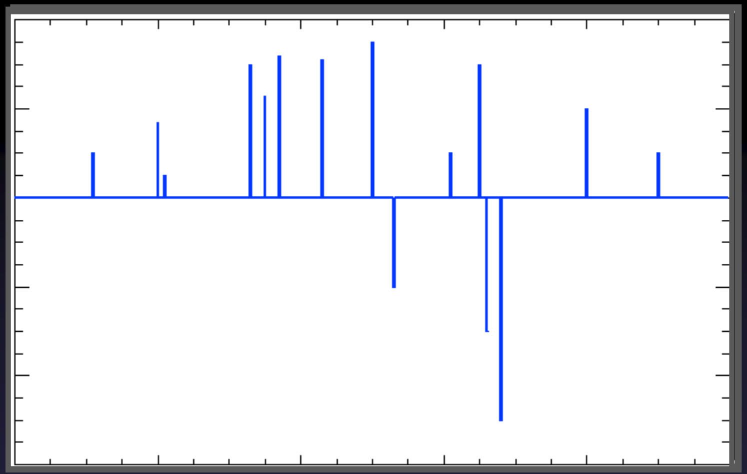
"Signals (x) with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements"

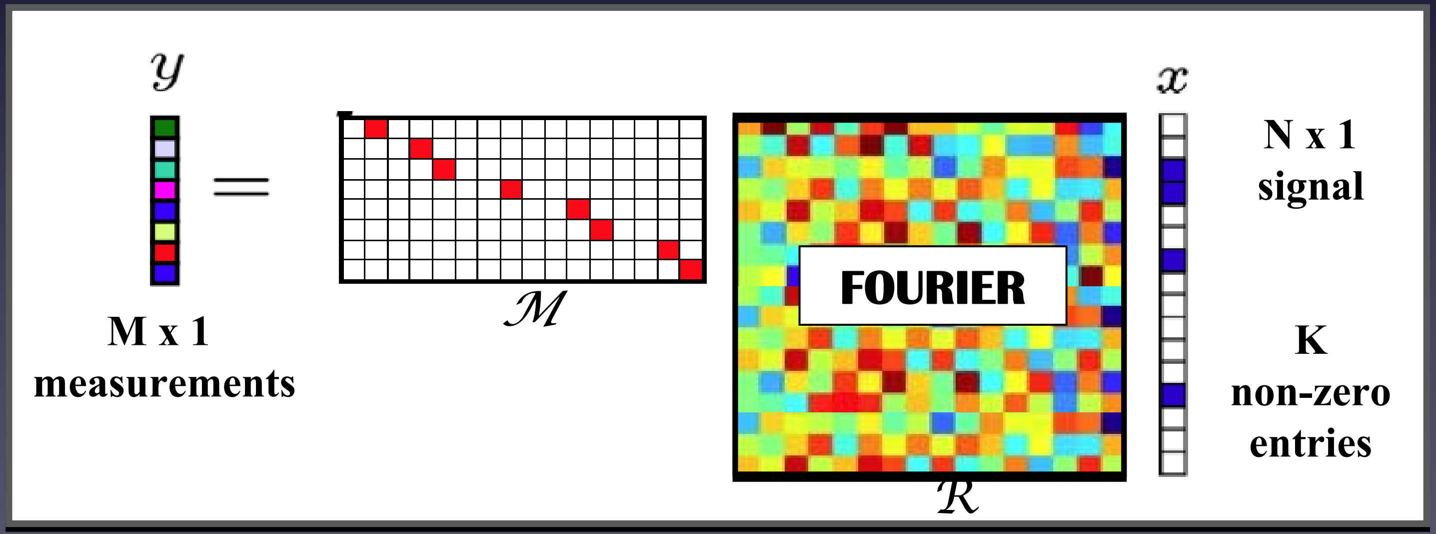
Replace samples by few linear projections $y = \Theta x = \mathcal{M}\mathcal{R}x$



Reconstruction via a non linear processing: $\min_{\alpha} ||\alpha||_1 \text{ s.t. } y = \Theta x$

Compressed Sensing

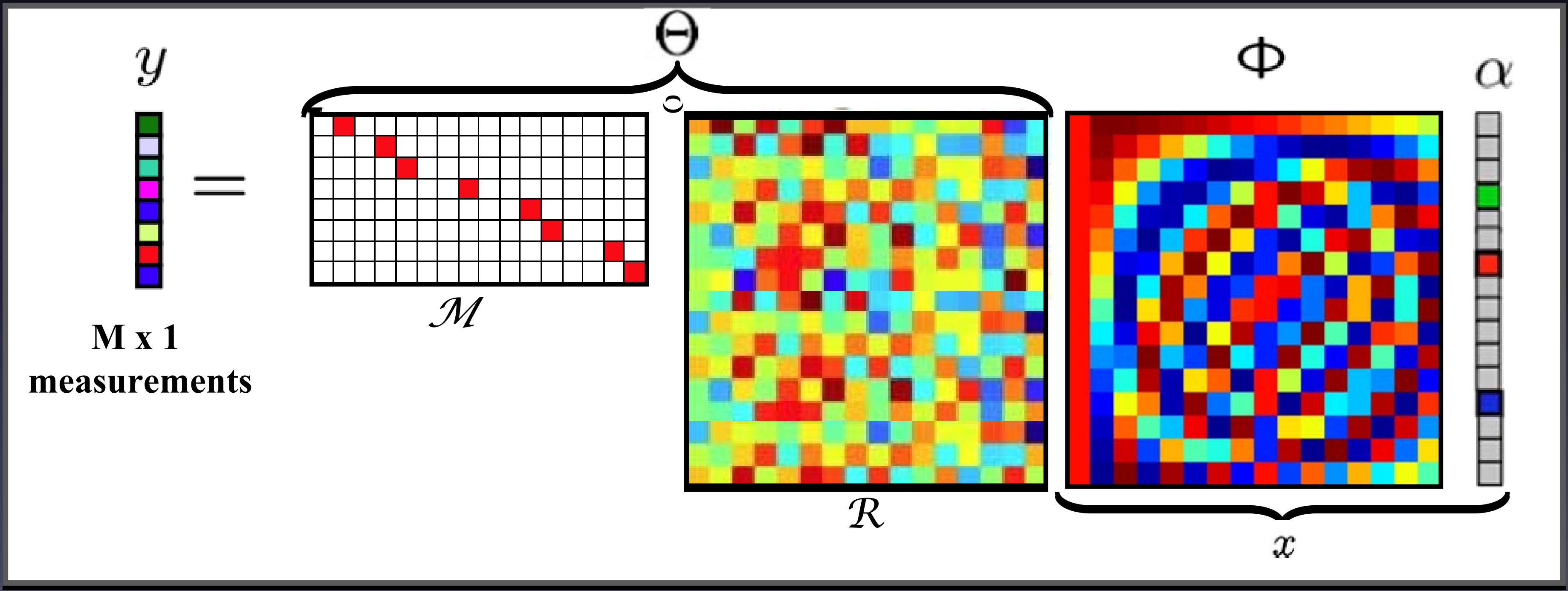




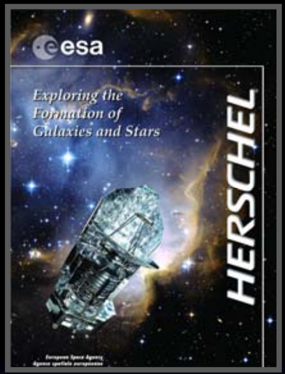
Soft Compressed sensing:

"Sparse signals (x) with exactly K coefficients (α) different from zero can be recovered perfectly from ~ K log N incoherent measurements"

$$y = \Theta x = \Theta \Phi \alpha$$



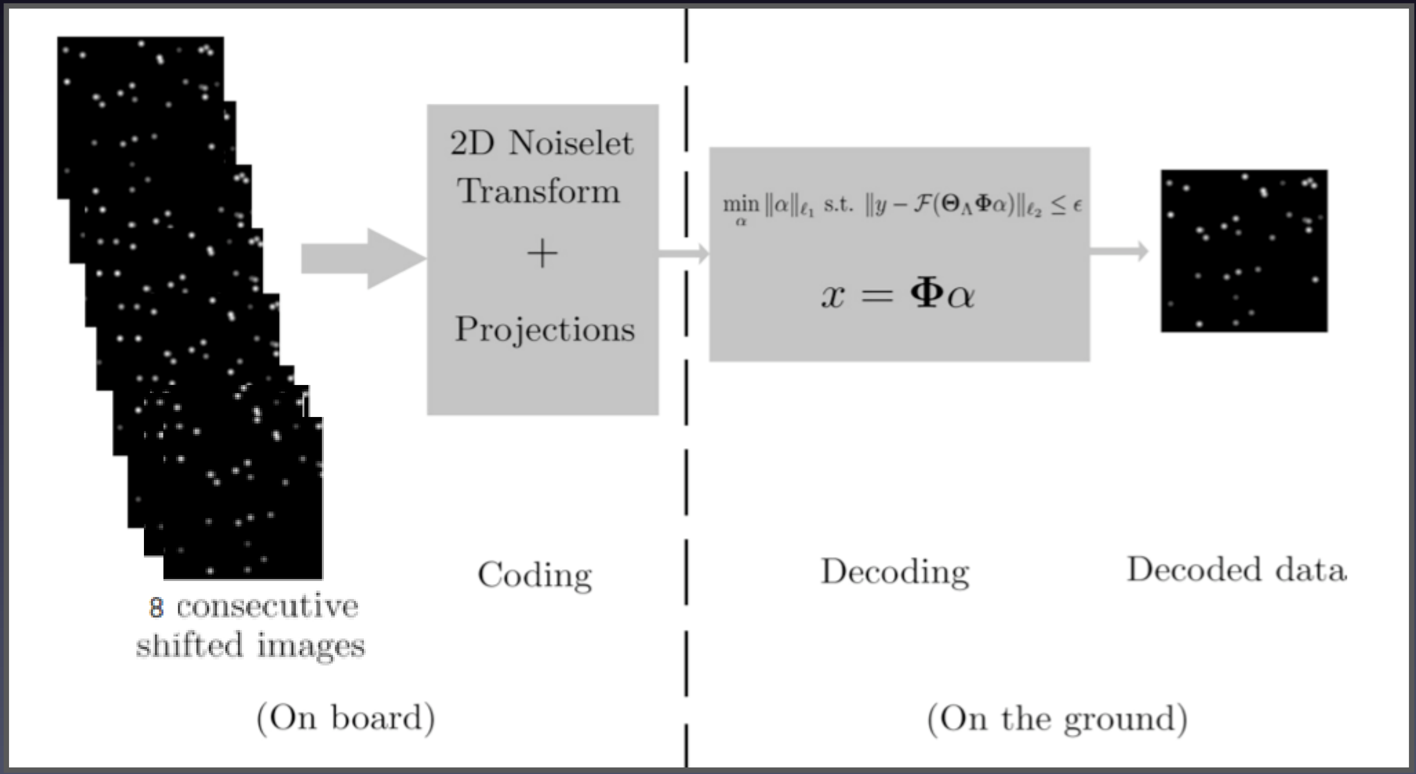
Reconstruction via a non linear processing: $\min_{\alpha} ||\alpha||_1 \text{ s.t. } y = \Theta \Phi \alpha$

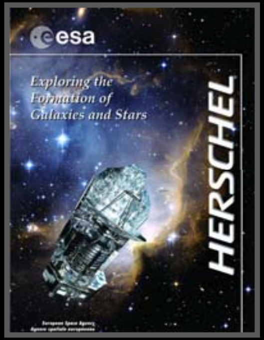


Compressed sensing to transfer spatial data to the earth

A field is obtained every 25 min and is composed by 60.000 shifted images (16x16 pixels)

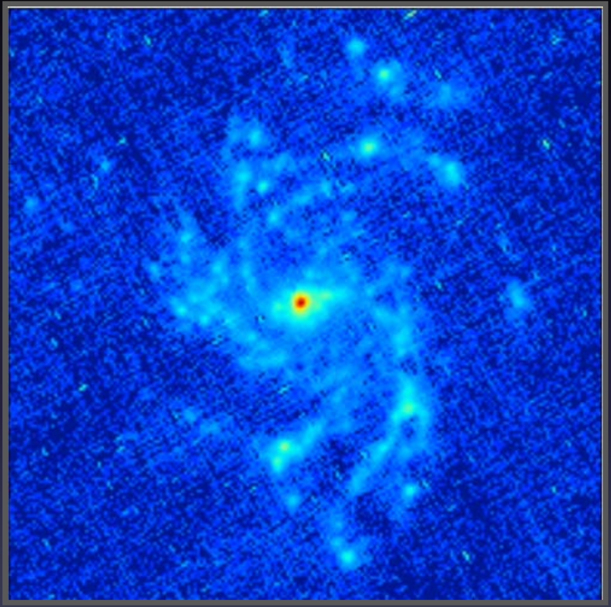
The official pipeline consists of only transmitting an averaged image obtained from 8 consecutives shifted images.



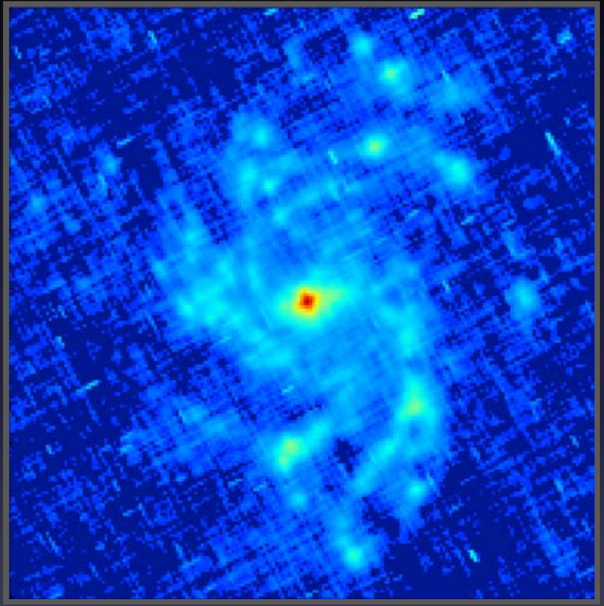


Compressed sensing to transfer spatial data to the earth

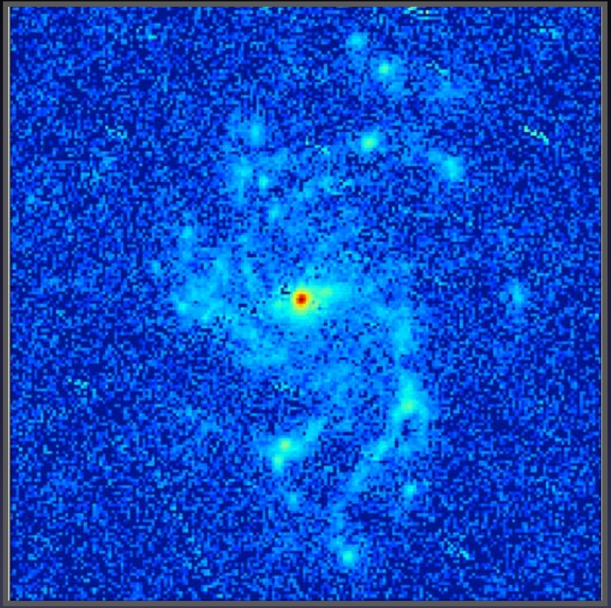
Good solution for on board data compression (very fast)



Map from uncompressed data



Official pipeline reconstruction: averaging



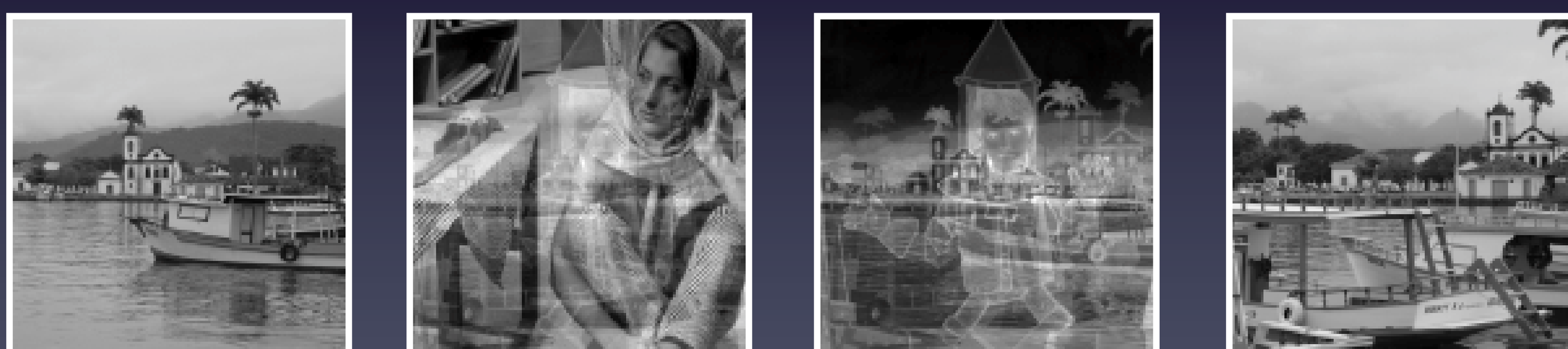
Compressed sensing reconstruction

Very robust to bit loss during transfer
All measurements are equally (un) important

Source Separation

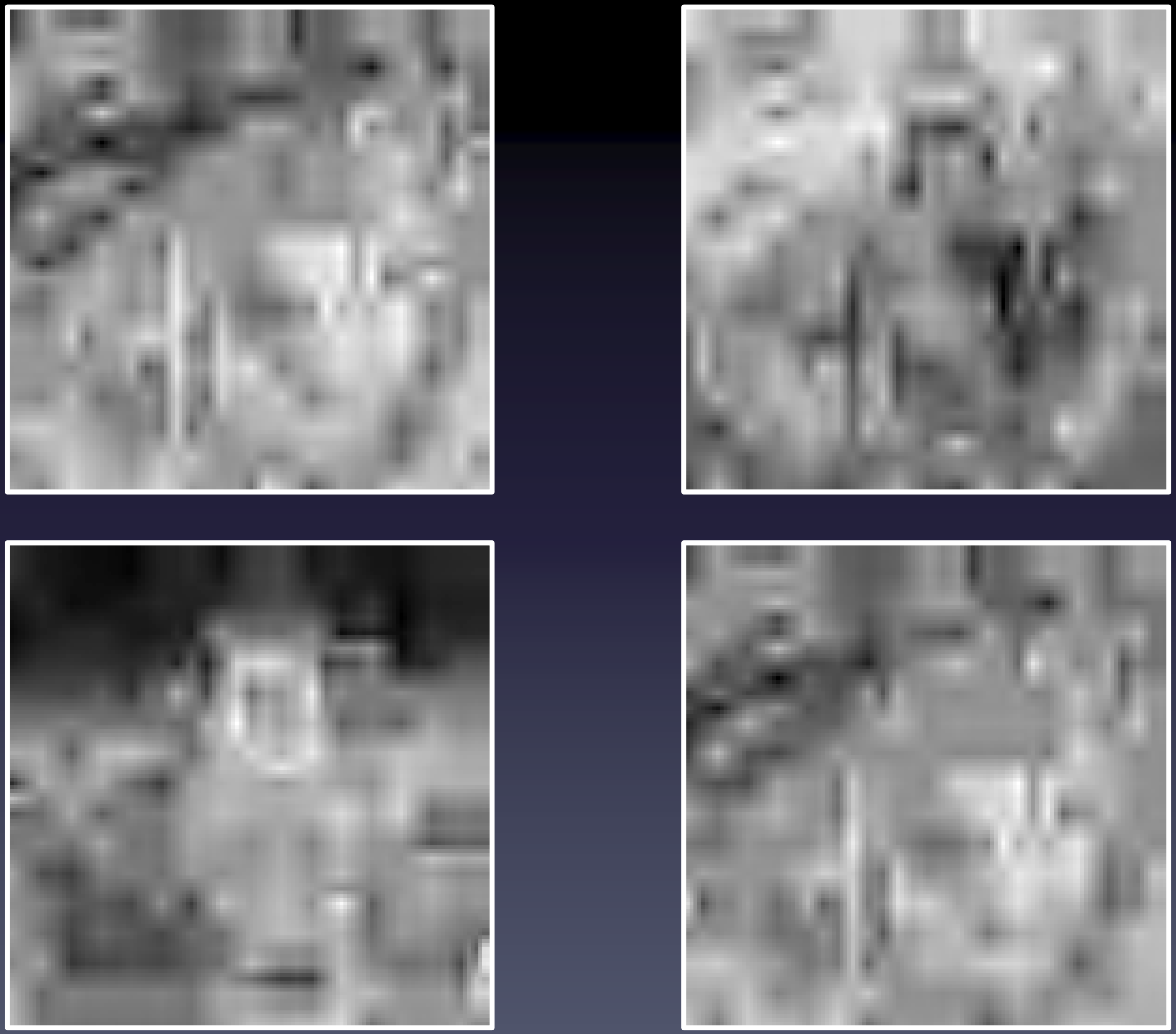
X = A S

4 sources

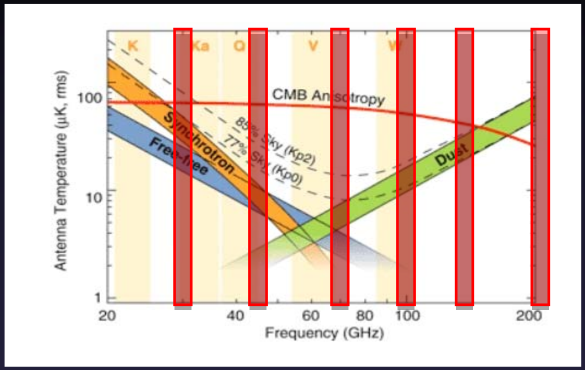


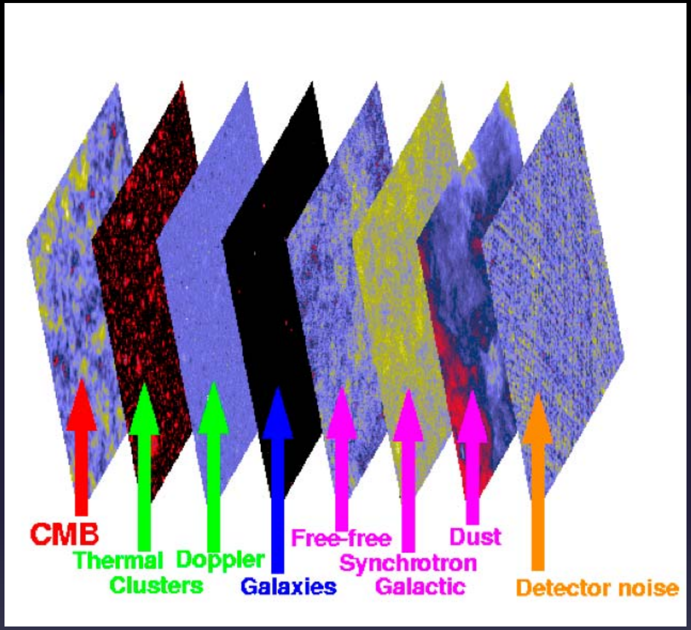
4 random mixtures

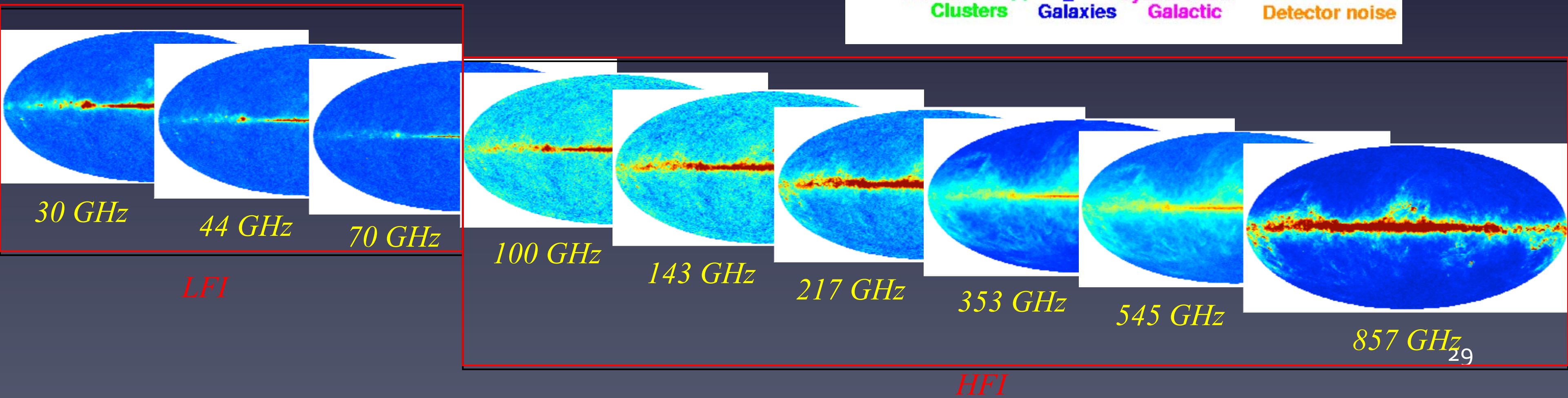
Source Separation



Source Separation: Cosmic Microwave Background

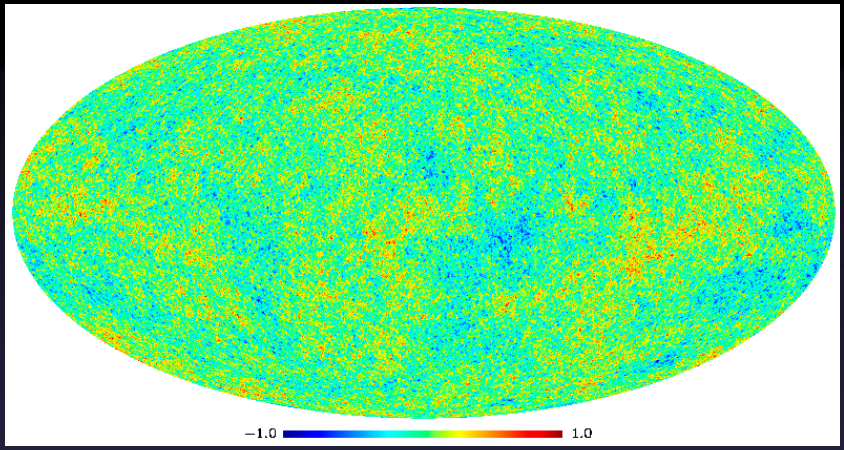




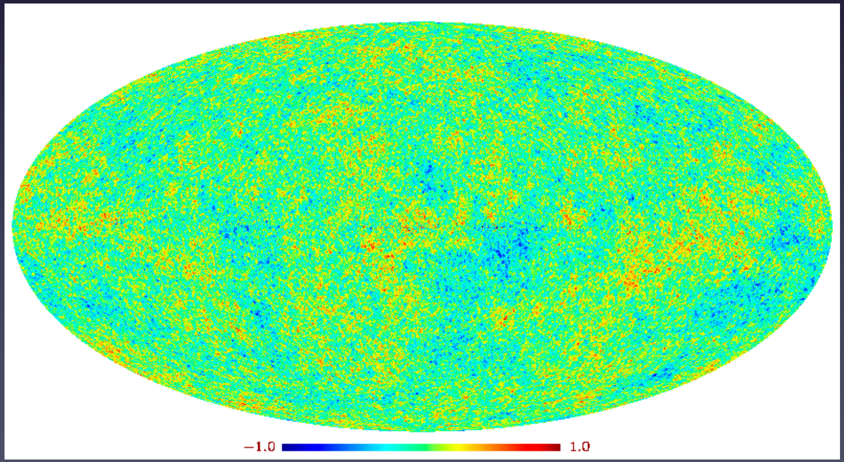


Source Separation: Cosmic Microwave Background

Input CMB map



CMB map estimated by GMCA



Conclusion

INSTRUMENTATION
The development of new instruments more and more accurate improves the quality of the observations

IMAGE PROCESSING
Improves the quality of the image after acquisition by the instrument

IMAGE PROCESSING
Is used to extract the useful information to help in the physical interpretation

END