Modeling crosstalk and afterpulsing in silicon photomultipliers

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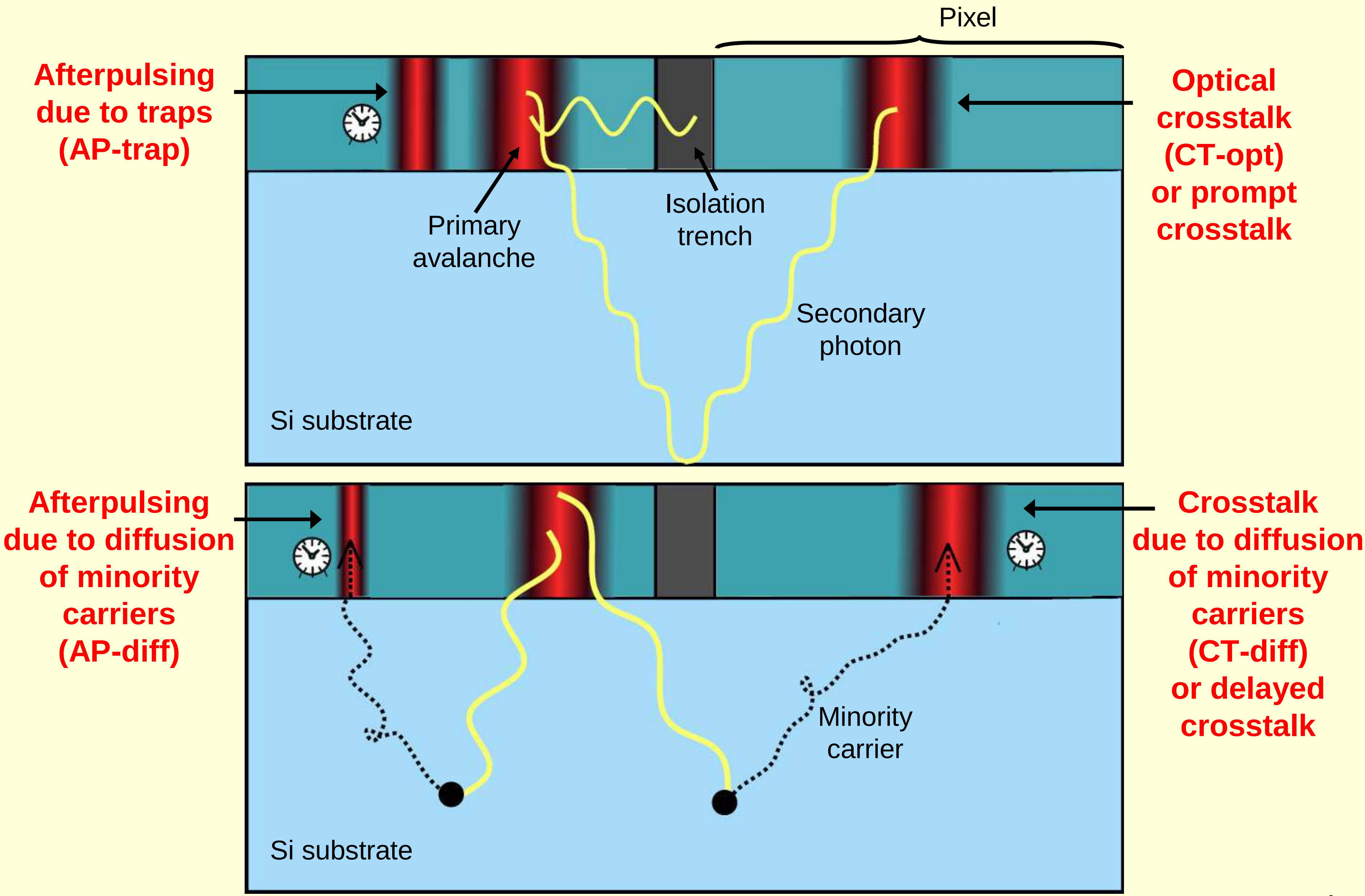
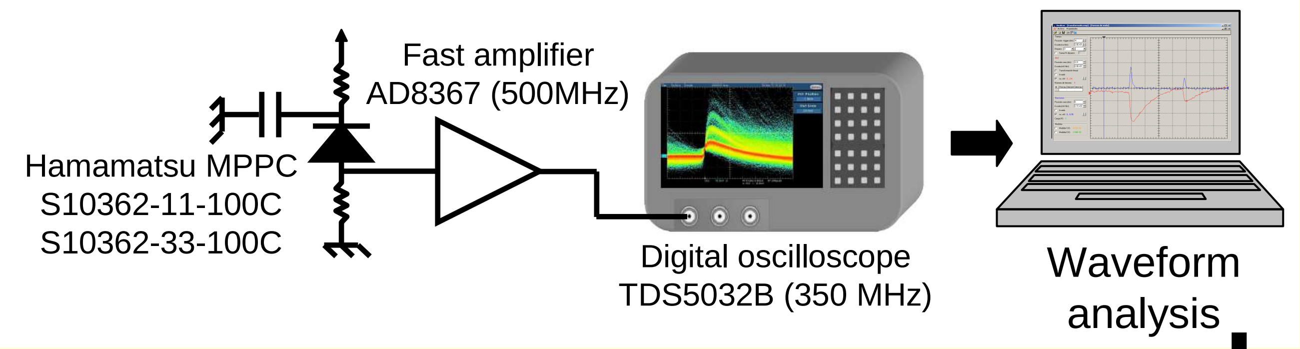
Definition of terms

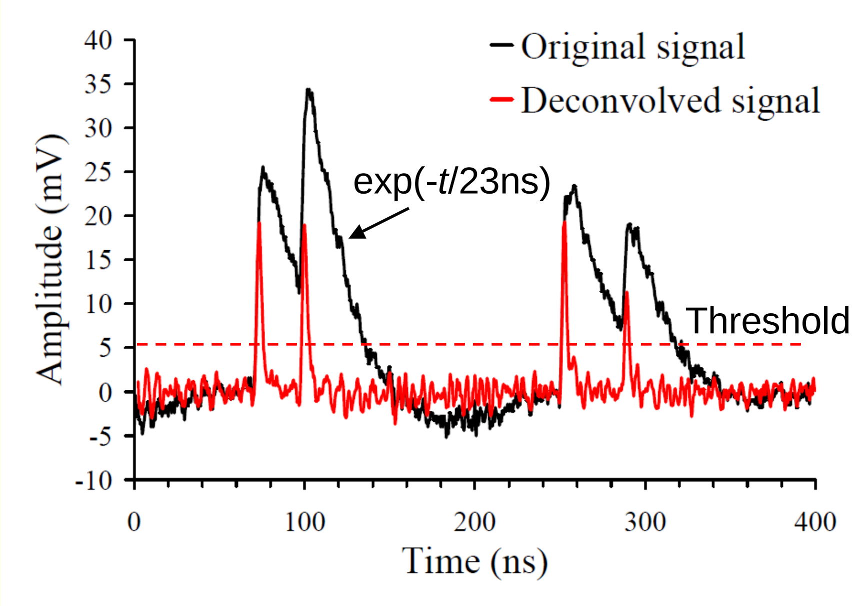


Experimental method

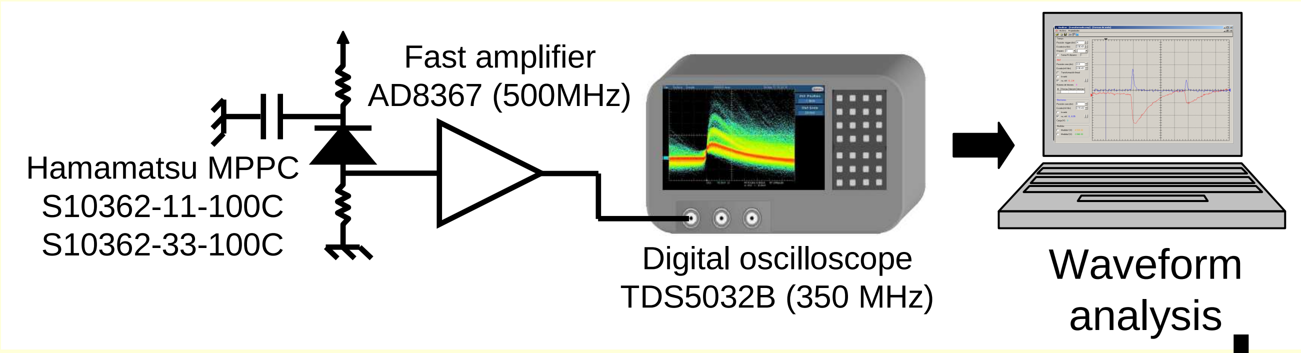


- Dark conditions
- Room temperature
- Nothing attached to the SiPM

- Pulse identification from **deconvolution**
- Arrival time with ~1 ns precision
- Close pulses distinguished with a resolution of ~6 ns
- Peak of deconvolution proportional to avalanche amplitude
- More precise measurement of amplitude as **pulse height with baseline subtraction**



Experimental method



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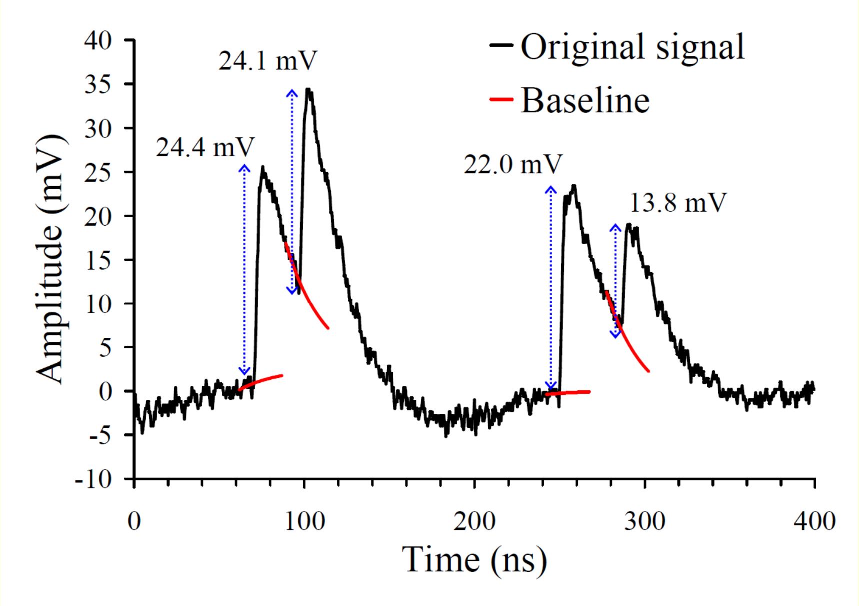
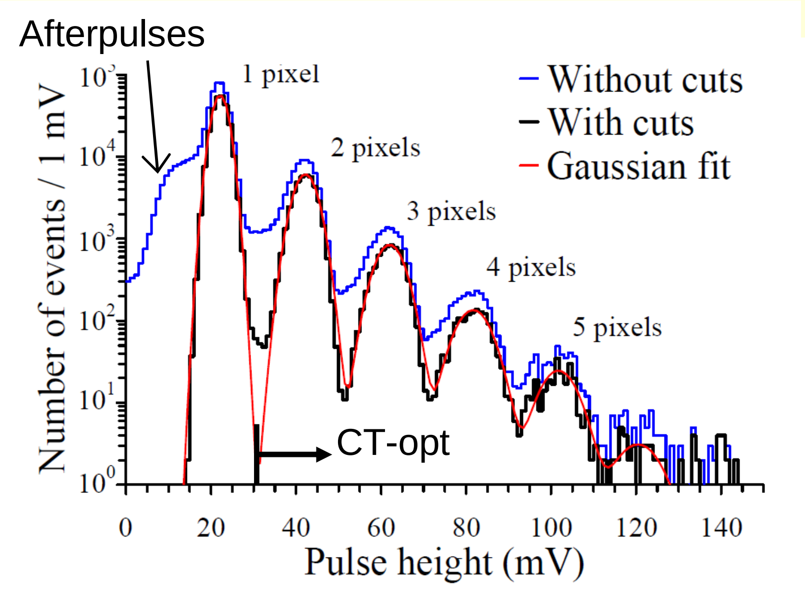
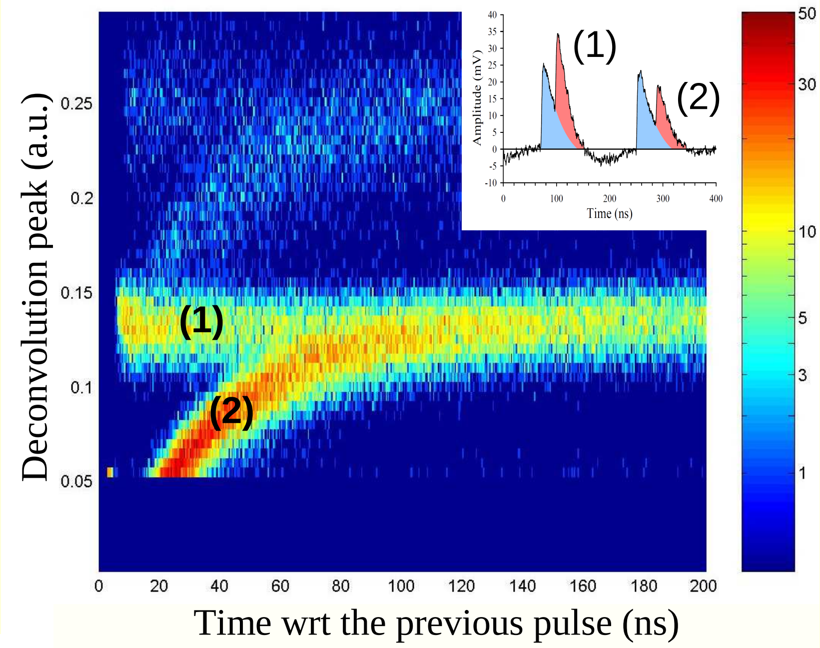
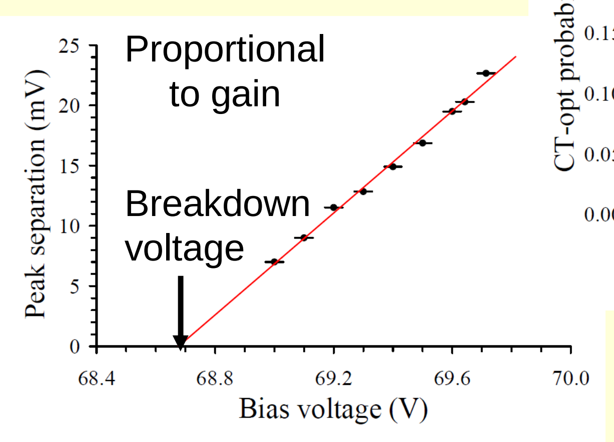
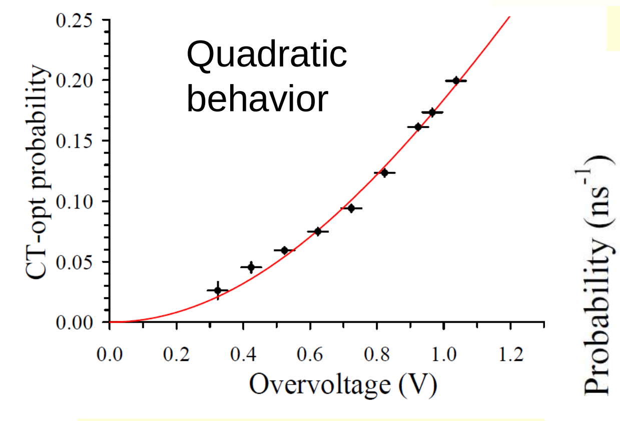


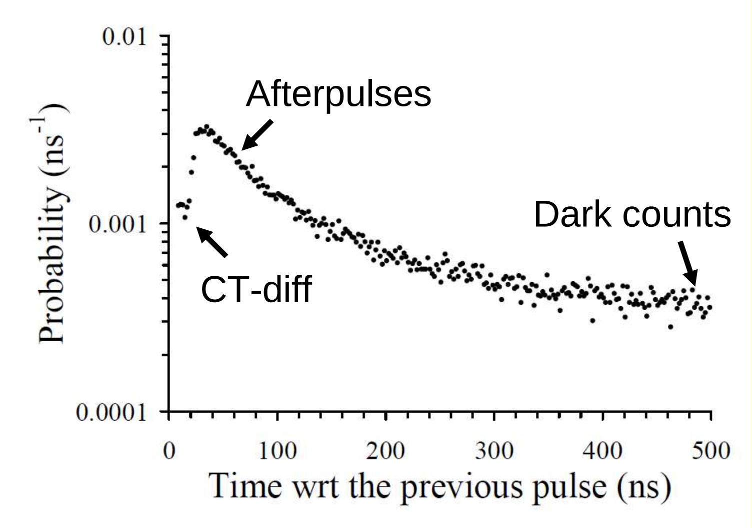
Experimental results











Modeling optical crosstalk: hypotheses

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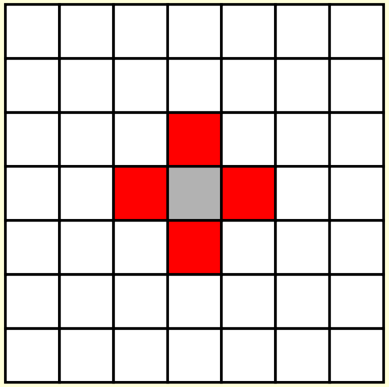
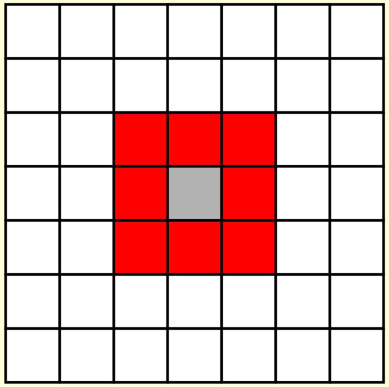
Probability distribution $P_1(k)$ (1 primary pixel + k-1 CT-opt)

- CT-opt only possible in a **neighborhood of pixels** around the primary one
- **Same probability** to excite any individual neighbor
- **Cascades** of CT excitations limited by **local saturation effects** (border effects ignored)

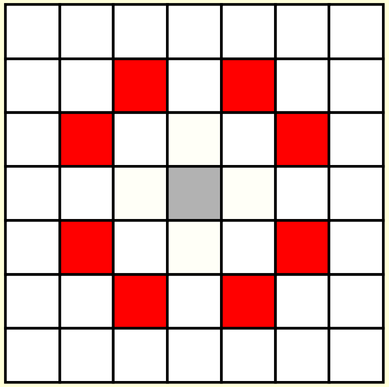
4 nearest neighbors



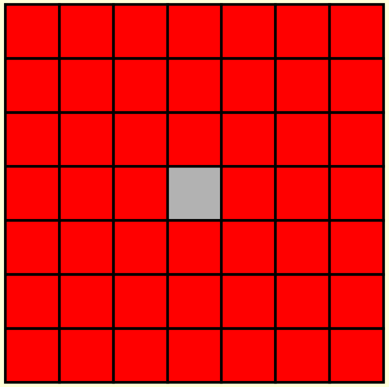
8 nearest neighbors

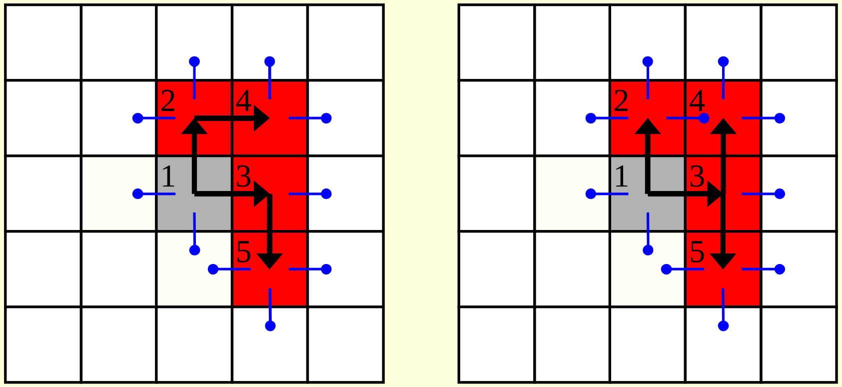


8 L-connected neighbors



All neighbors





Example for the 4 nearest neighbors: Two different CT-opt “histories” contributing to $P_1(5)$

Modeling optical crosstalk: formulations

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Analytical expressions for P_1(k) and related parameters

k	4 nearest neighbors	8 nearest neighbors	8 L-connected neighbors	All neighbors
1	$q^4(=1-\varepsilon)$	$q^8(=1-\varepsilon)$	$q^8(=1-\varepsilon)$	$q^{N-1}(=1-\varepsilon)$
2	$4pq^6$	$8pq^{14}$	$8pq^{14}$	$\binom{N-1}{1} pq^{2(N-2)}$
3	$18p^2q^8$	$12p^2q^{18}[1+2q+4q^2]$	$84p^2q^{20}$	$\binom{N-1}{2} p^2q^{3(N-3)}[1+2q]$
4	$4p^3q^8[1+3q+18q^2]$	$4p^3q^{20}[1+3q+14q^2+30q^3+61q^4+59q^5+72q^6]$	$24p^3q^{24}[1+3q+38q^2]$	$\binom{N-1}{3} p^3q^{4(N-4)}[1+3q+6q^2+6q^3]$
5	$5p^4q^{10}[8+24q+55q^2]$	$5p^4q^{24}[9+36q+98q^2+188q^3+310q^4+372q^5+520q^6+396q^7+341q^8]$	$4p^4q^{30}[180+540q+2521q^2]$	$\binom{N-1}{4} p^4q^{5(N-5)}[1+4q+10q^2+20q^3+30q^4+36q^5+24q^6]$

p : prob. for 1 neighbor $q = 1-p$ $\varepsilon = P_1(k>1)$ *N* : number pixels of the array

Geometric extrapolation for *k* > 5

$$P_1(k) \approx P_1(5)\cdot(1-r)^{k-5}$$

$$r = \frac{P_1(5)}{1-\sum_{k=1}^{4} P_1(k)}$$

$$E_1 \approx \sum_{k=1}^{4} k\cdot P_1(k) + P_1(5)\frac{1+4r}{r^2}$$

$$\mathrm{Var}_1 \approx \sum_{k=1}^{4} k^2\cdot P_1(k) + P_1(5)\frac{2+7r+16r^2}{r^3} - E_1^2$$

$$\mathrm{ENF} = \frac{\sum_{k=1}^{4} k^2\cdot P_1(k) + P_1(5)\frac{2+7r+16r^2}{r^3}}{\left[\sum_{k=1}^{4} k\cdot P_1(k) + P_1(5)\frac{1+4r}{r^2}\right]^2}$$

Validation with data at dark conditions

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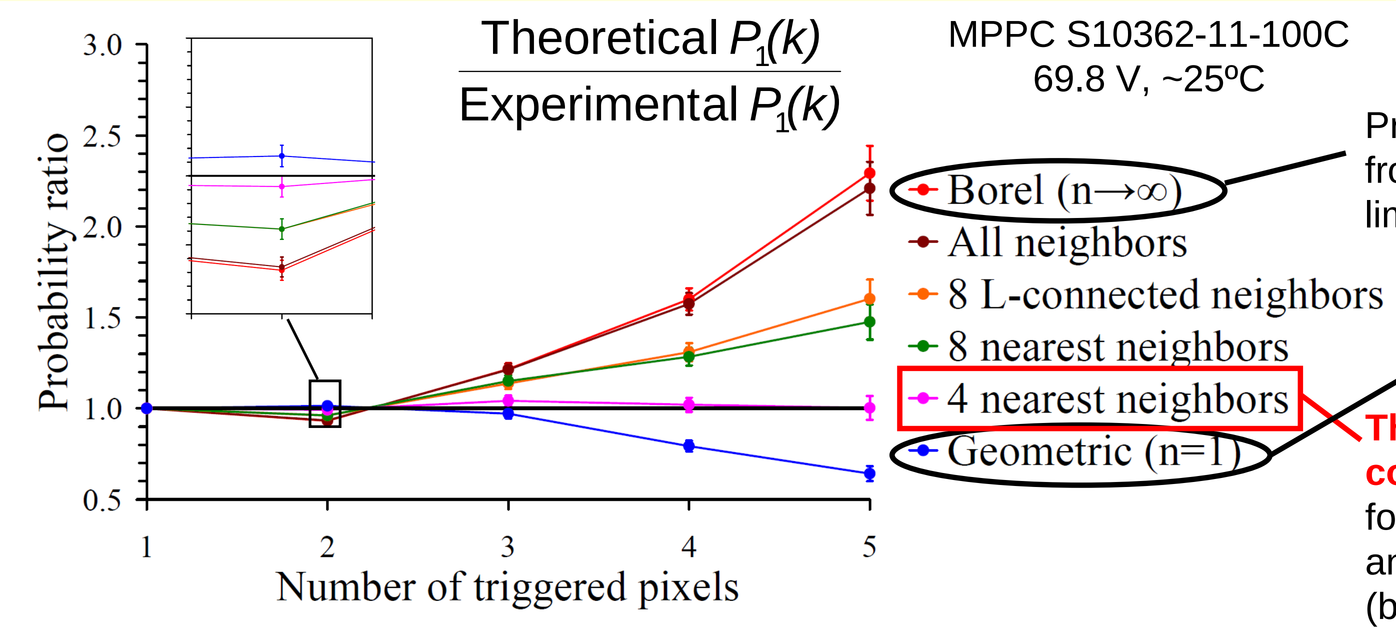
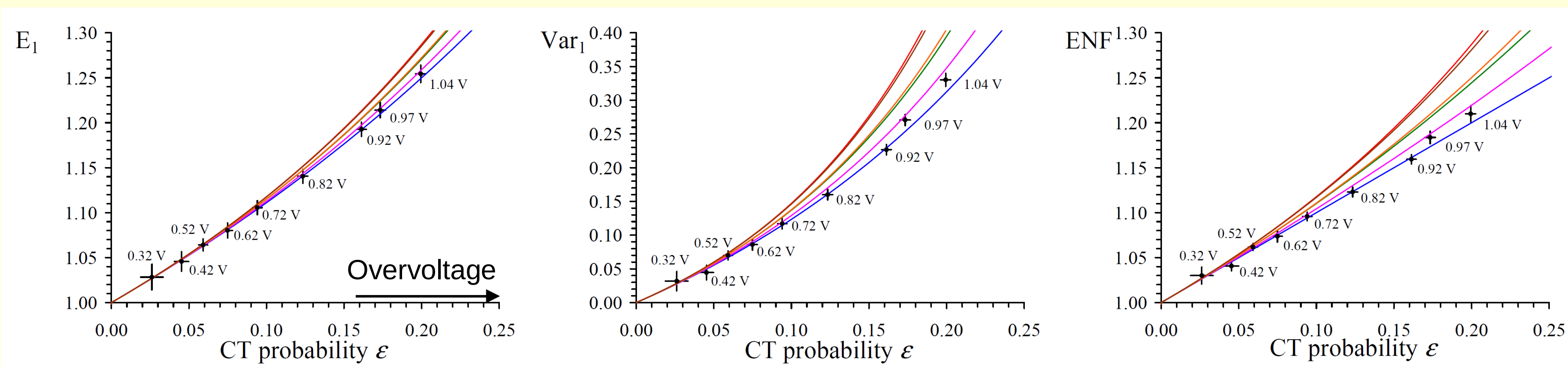


Previous analytical models from S. Vinogradov: limit situations of ours

The only model consistent with our data for both s10365-11-100C and s10365-33-100C (border effects unimportant)



$\varepsilon = P_1(k>1) = 1 - P_1(1)$

Application to photon counting

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Under pulsed illumination, i.e., **simultaneous incoming photons**

$P_{ph}(k)$: prob. distribution of real detected photons
$P_{obs}(k)$: observed prob. distribution (with CT-opt)
$\varepsilon = P_1(k>1)$: overall CT-opt probability

$$P_{ph}(0) = P_{obs}(0), \quad P_{ph}(1) = \frac{P_{obs}(1)}{1-\varepsilon}$$

$$P_{ph}(k) = \frac{1}{(1-\varepsilon)^k}\left[P_{obs}(k) - \sum_{n=1}^{k-1} P_{ph}(n)\cdot P_n(k)\right], k = 2,3,4,\ldots$$ Recursive

$$P_{n+1}(k) = \sum_{i=1}^{k-n} P_n(k-i)\cdot P_1(i), \quad n = 1,2,3,\ldots$$

Recursive

$P_1(k)$ calculated for dark conditions

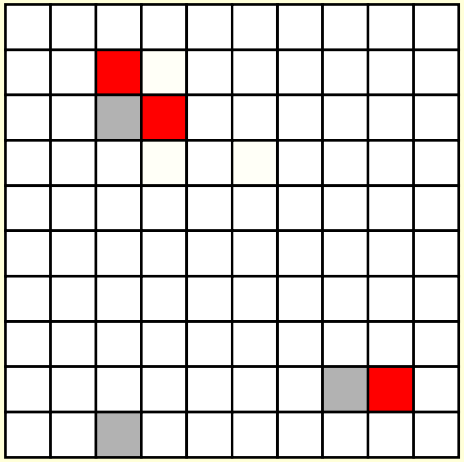
Mean and variance of real number of photons:

$$E_{obs} = E_{ph}\cdot E_1$$

$$Var_{obs} = E_{ph}\cdot Var_1 + Var_{ph}\cdot E_1^2$$

E_1 and Var_1 are known

Approximation valid for k << total number of pixels, i.e., **linear range**



Modeling AP and CT-diff: method

Distribution of arrival time wrt the previous pulse (primary). We select primaries with amplitudes of 1 pixel and far from former pulses (> 500 ns)

- Poisson statistics for each source of secondary pulses
- **2 components of AP-trap** with different mean release time
- **Timing** of carrier diffusion and the relative contributions of **AP-diff and CT-diff obtained by Monte Carlo**
- **Pixel recovery and detection threshold effects** included for AP

$$P(t)\mathrm{d}t = \exp\left\{ -R_{\mathrm{DC}}(t - t_{\min}) - \sum_i \lambda_i \int_{t_{\min}}^{t} f_i(s)\mathrm{d}s \right\}$$

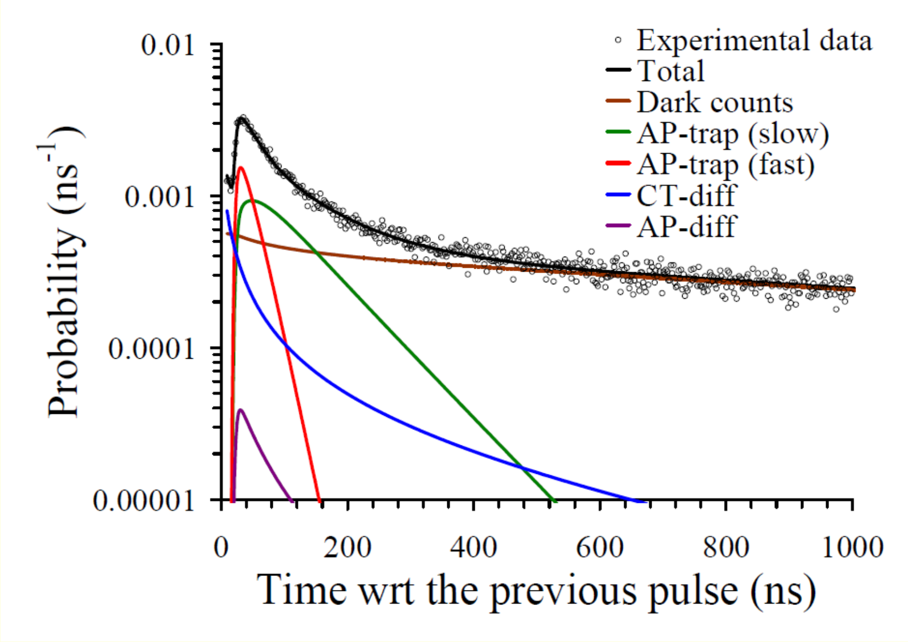
$$\times \left[R_{\mathrm{DC}} + \sum_i \lambda_i f_i(t) \right] \mathrm{d}t$$

$t_{\min}$ = 10 ns : minimum t (analysis limitations)

R_{DC} : dark count rate

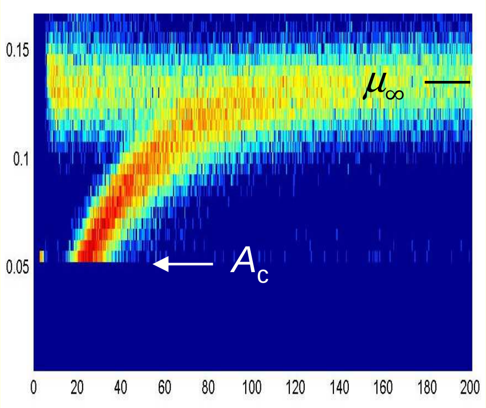
λ_i : average number of secondary pulses of type i per primary avalanche

$f_i(t)$: normalized time distribution of secondary pulses of type i



Effective time distribution of afterpulses

For each AP-trap component



$$f(t) \propto \exp\left(-\frac{t}{\tau}\right)\mu(t)k(t)$$

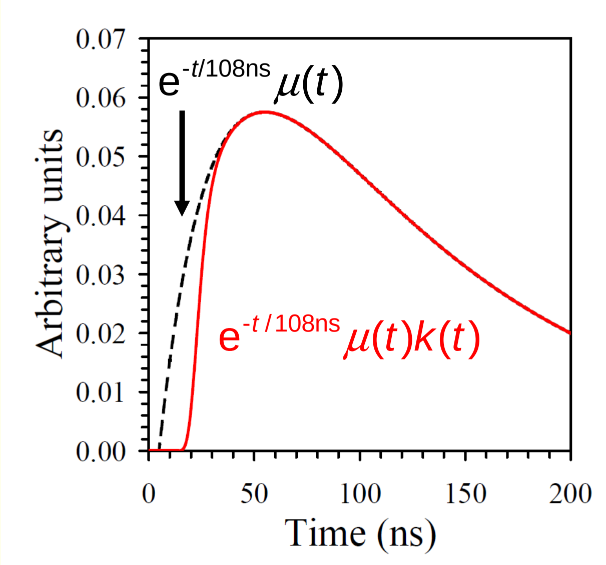
τ: mean release time of trapped carriers

$$\mu(t) = \mu_\infty \theta(t - t_0)\left[1 - \exp\left(-\frac{t - t_0}{t_{rec}}\right)\right]$$

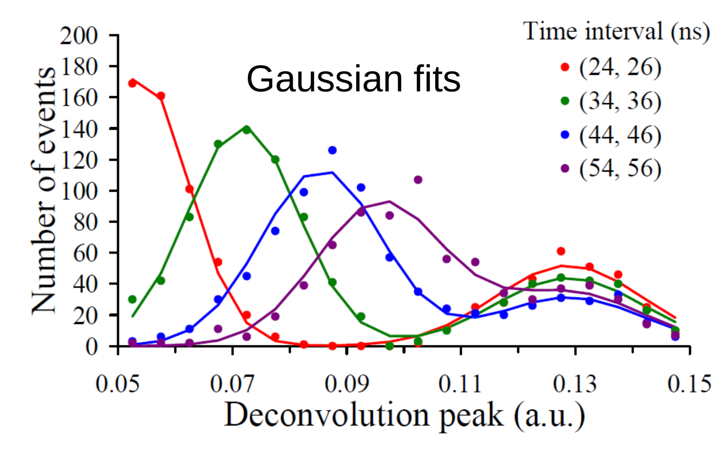
Average AP amplitude
∝ pixel recovery $V(t)$
∝ avalanche probability

$$k(t) = \frac{1}{\sqrt{2\pi C\mu(t)}}\int_{A_c}^{\infty}\exp\left[-\frac{(A - \mu(t))^2}{2C\mu(t)}\right]dA$$

Fraction of APs above threshold A_c



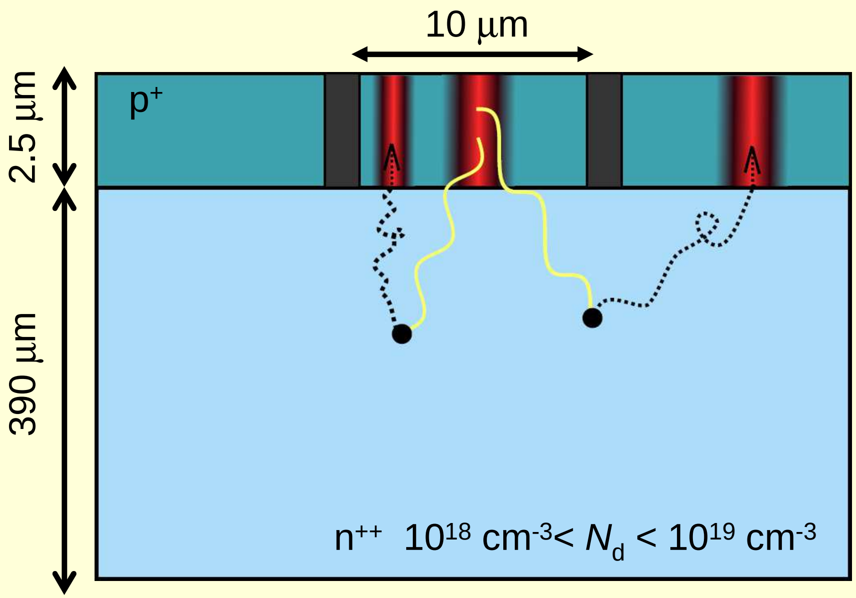
AP amplitudes at given t are assumed to follow a Gaussian distribution with mean $\mu(t)$ and variance $\sigma^2(t) = C\cdot\mu(t)$



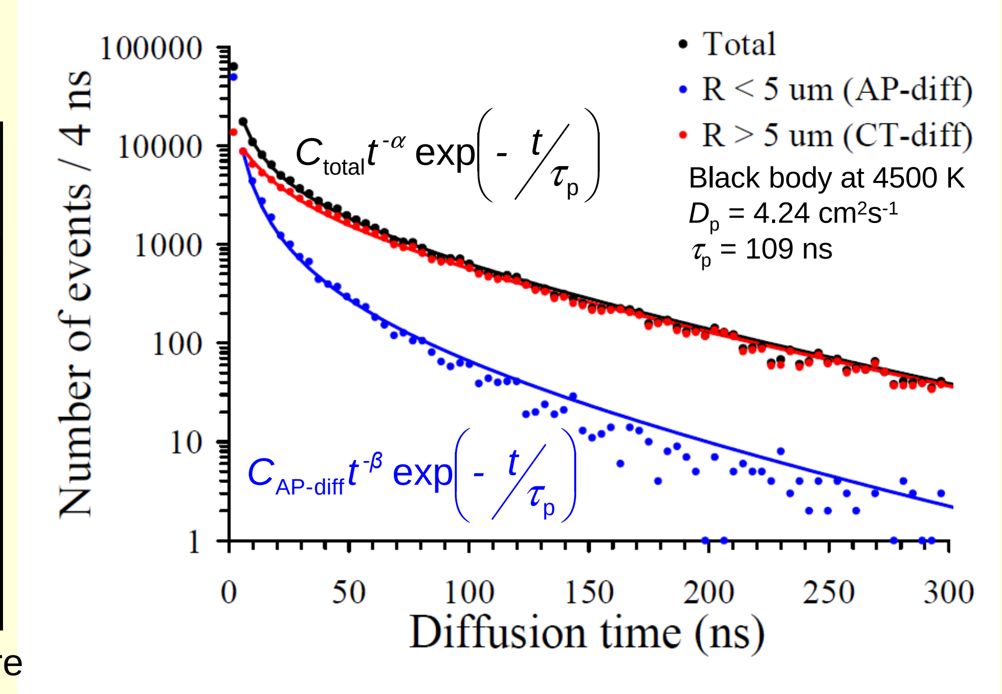
Fitting parameters:
μ_∞, t_0, t_{rec}, C

$t_0 \approx 5$ ns , $t_{rec} \approx 37$ ns

Monte Carlo of carrier diffusion



Necessary parameters taken from the literature with reasonable range of variation



τ_p : lifetime of minority carriers (p) in Si n^{++} (before being recombined)

Time distributions:

$$f_{AP\text{-}diff}(t) \propto C_{AP\text{-}diff} t^{-\beta} \exp\left(-\frac{t}{\tau_p}\right) \overbrace{\mu(t)k(t)}^{\text{same as AP-trap}}$$

$$f_{CT\text{-}diff}(t) \propto \left[C_{total} t^{-\alpha} - C_{AP\text{-}diff} t^{-\beta}\right] \exp\left(-\frac{t}{\tau_p}\right) \mu_\infty$$

Relative contributions:

$$\left.\begin{array}{l} C_{AP\text{-}diff} \sim C_{total} \\ \mu(t)k(t) << \mu_\infty \quad (t < 20 \text{ ns}) \end{array}\right\} \lambda_{AP\text{-}diff} << \lambda_{CT\text{-}diff}$$

Results at given voltage

- **2 AP-trap components** actually needed. The slow one is dominant

  τ_{slow} = 108 ± 30 ns ; λ_{slow} = 0.170 ± 0.022

  τ_{fast} = 23 ± 5 ns ; λ_{fast} = 0.065 ± 0.020

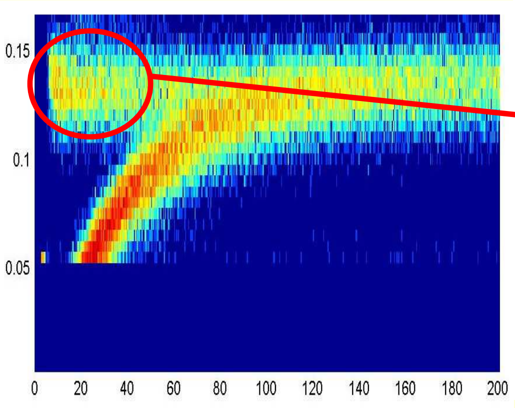
- **CT-diff also important** at short time

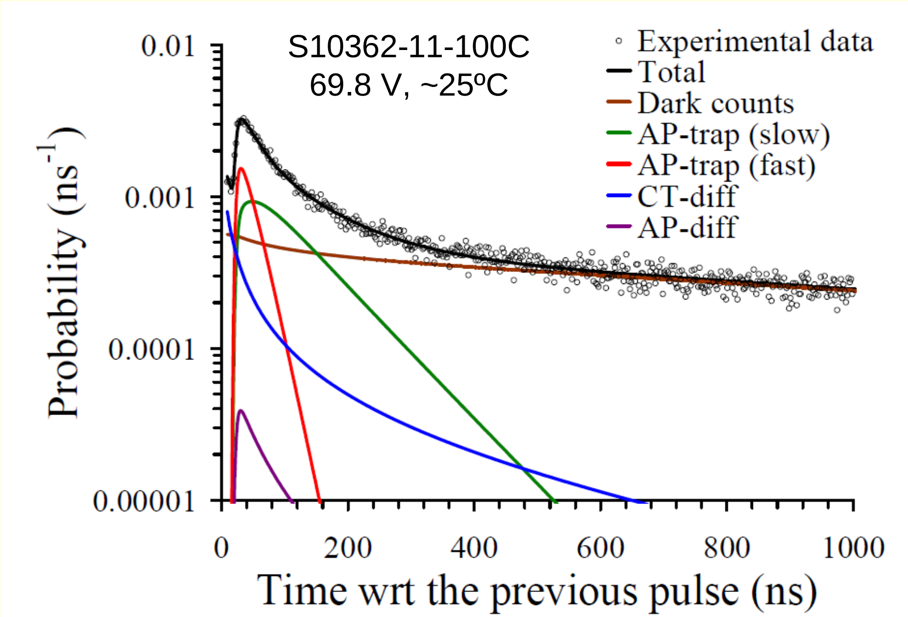
  $\lambda_{CT\text{-}diff}$ = 0.062 ± 0.028

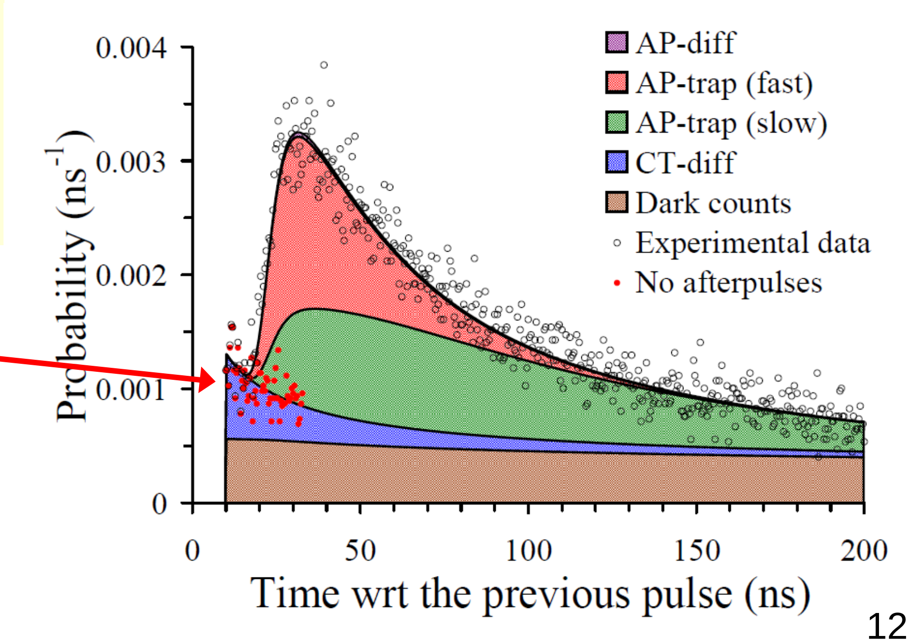
  $\lambda_{AP\text{-}diff}$ < 0.012

- Best fit for $\tau_p \sim 1\ \mu s$ ($N_d \sim 10^{18}\ cm^{-3}$), but **not too much sensitive to the parameters implemented in the simulation**

At short time dark counts and CT-diff can be separated from AP

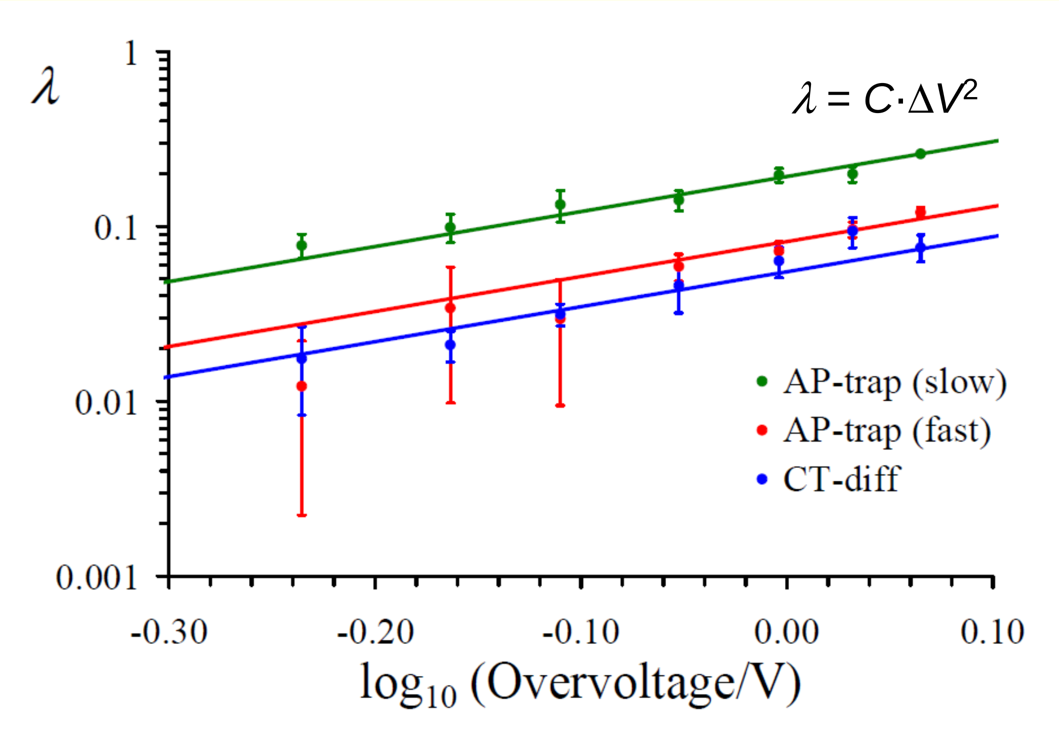


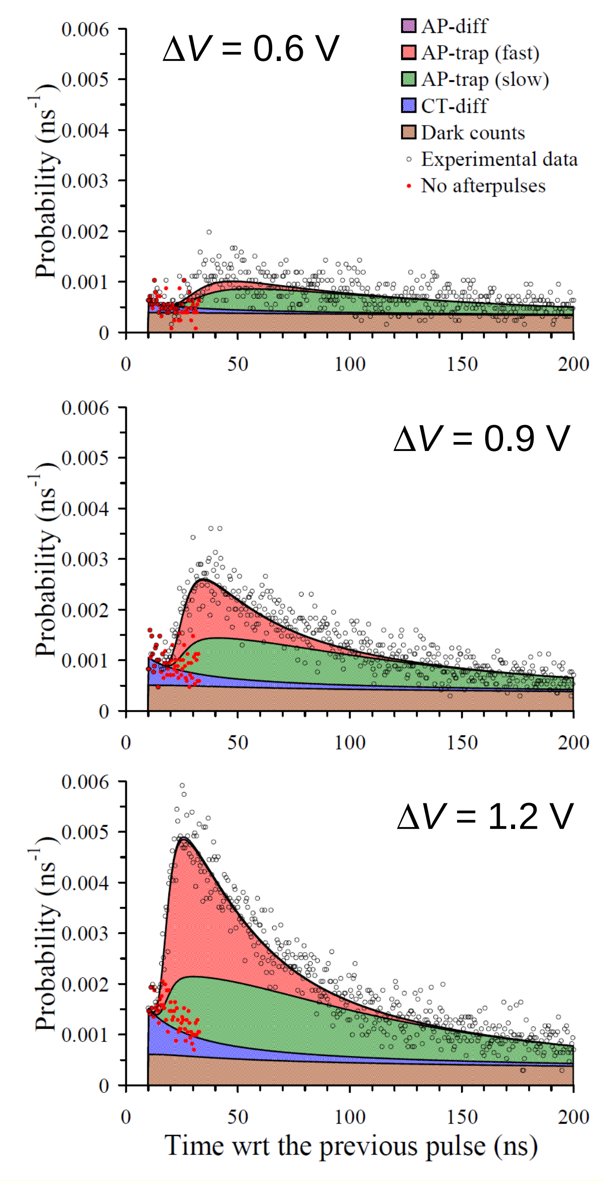




Dependence on overvoltage

- **Consistency in the time distributions** $f_i(t)$ as a function of overvoltage ΔV. In particular, constant mean release time for both AP-trap components (i.e., τ_{slow} and τ_{fast}).
- **Number of secondary pulses** of each type λ_i (~ probability) **grows quadratically** as expected





Conclusions I

- We have developed an experimental method based on a waveform analysis to characterize CT and AP in SiPMs

Optical crosstalk:

- We constructed a statistical, analytical model taking into account:
  - Pixels have a finite number of neighbors
  - Cascades of CT excitations
  - Saturation effects due to pixel dead time
- Experimental data (S10362-11-100C and S10362-33-100C from Hamamatsu) are consistent with the hypothesis that CT-opt only takes place between adjacent pixels
- Correction for CT-opt effects on photon counting measurements at pulsed illumination of low intensity

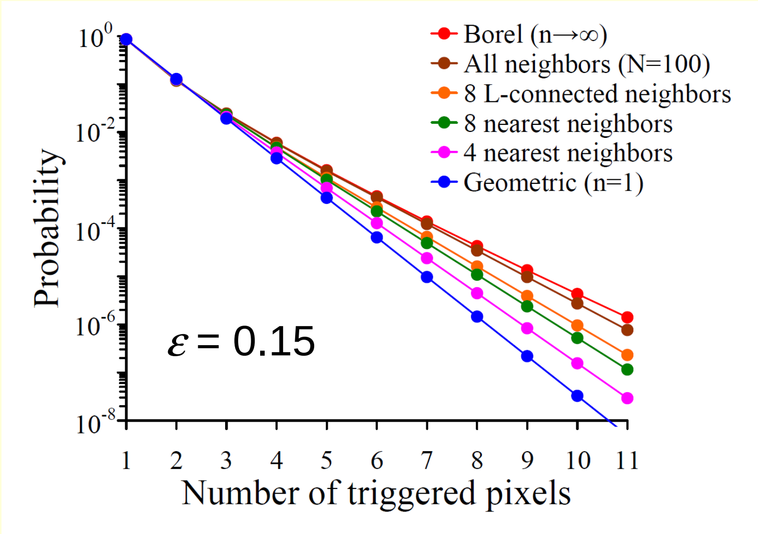
Conclusions II

Afterpulsing and delayed crosstalk

- We constructed a statistical model based on:
  - Poisson statistics for number of secondary pulses
  - 2 types of traps with different mean release time
  - Timing of carrier diffusion and relative probabilities of AP-diff and CT-diff determined by Monte Carlo
  - Pixel recovery time and threshold effects included
- Slow component of AP-trap is dominant but the fast one and CT-diff are significant too
- AP and CT-diff probabilities grow quadratically with overvoltage
- We have not applied the model to any experimental case yet, but it is suitable to be implemented in a Monte Carlo simulation of an experiment to account for these effects

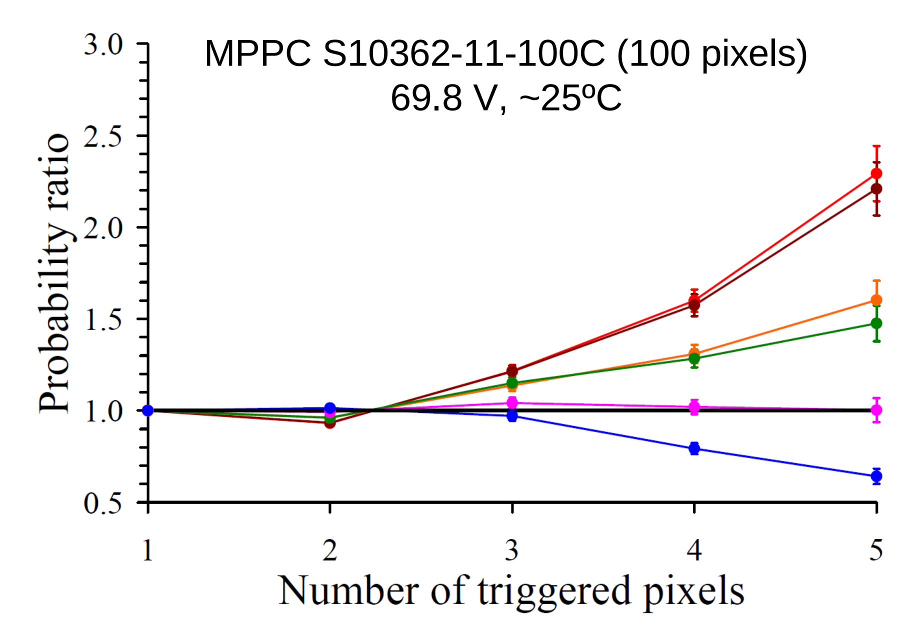
THANK YOU

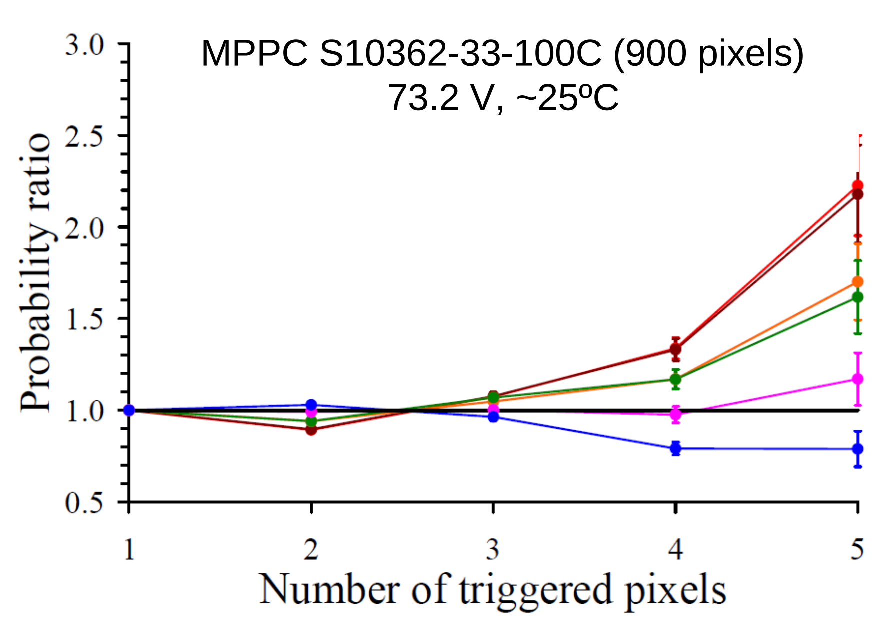
Backup



Theoretical probability distributions

Results independent of the size of the array
→ border effects are unimportant

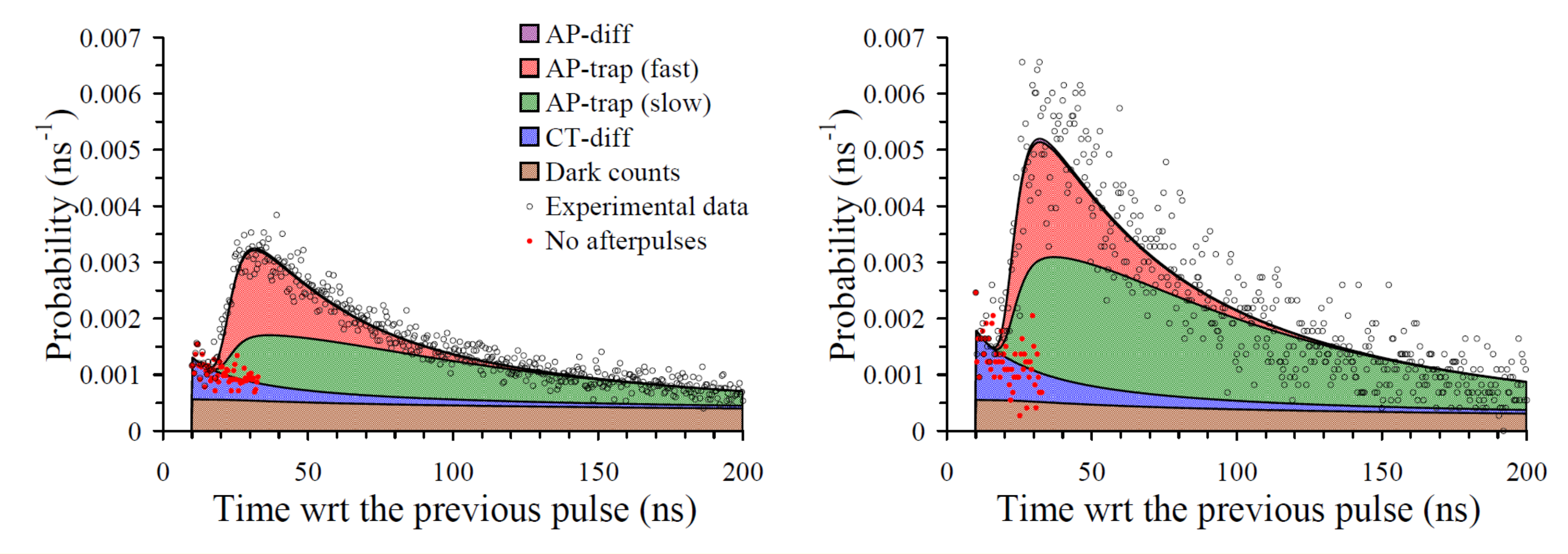




Backup

Selecting primary pulses with amplitudes of 1 pixel

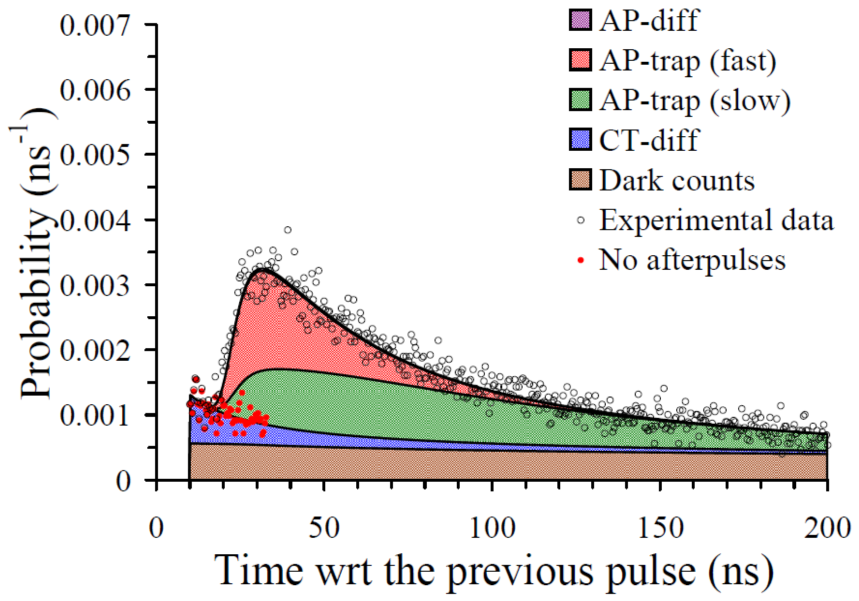
Selecting primary pulses with amplitudes of **more than 1 pixel**



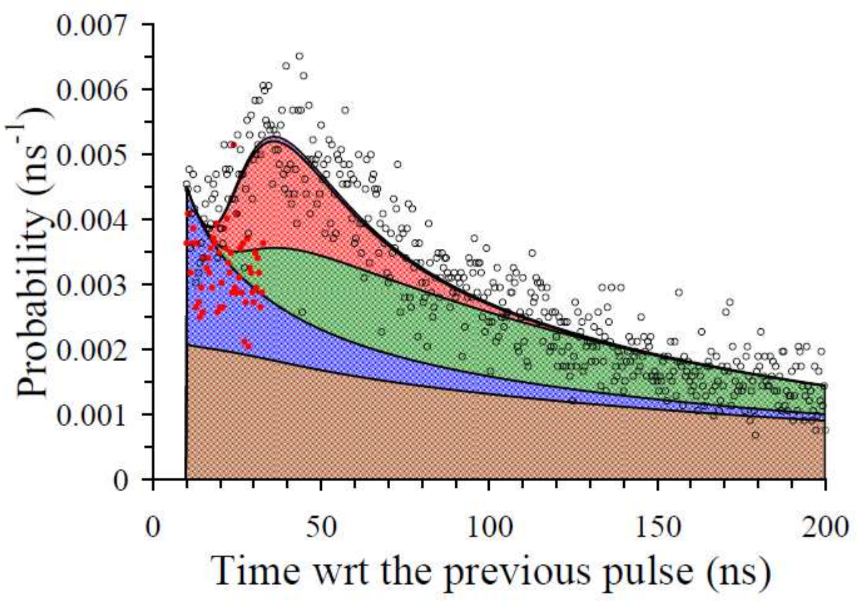
Probabilities of AP and CT-diff increase by a factor ~2 as expected

Backup

MPPC S10362-11-100C (100 pixels)
69.8 V, ~25ºC



MPPC S10362-33-100C (900 pixels)
73.2 V, ~25ºC



Consistency in the time distributions
More dark counts
Higher CT-diff probability