Impact of the Readout Modes, Signal Fitting and Time Sampling on the total noise of a H2RG near-IR detectors

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Understand the Geometry of the Universe with 2 cosmological probes:

**Weak Gravitational Lensing and Baryon Acoustic Oscillations**

- **Dark Matter** - CDM paradigm for structure formation, sum of the $\nu$ masses to a precision better than 0.04 eV when combined with Planck.
- **Dark Energy** - expansion history and structure growth (DE equation of state parameters measured to a precision of 2%)
- **Gravity** - GR vs modified-gravity theories (growth rate exponent $y$ with a precision of 2%)
- **Wide survey:** $> 15,000 \text{ deg}^2$ (36% of the total sky)
- **Deep survey:** $> 40 \text{ deg}^2$ (2 mag deeper than wide survey)
- **Weak Lensing:** shapes and shear of $> 30 \text{ galaxies/arcmin}^2$ for $0 < z < 2$, accuracy $dz/z \sim 0.04 \rightarrow$ very high image quality and stability (ellipticity, FWHM, R2) systematic $\sigma_{sys} < 10^{-7}$
- **Galaxy clustering:** redshifts for $> 3500 \text{ galaxies/deg}^2$ in the range $0.7 < z < 2.05$ with accuracy $dz/z < 0.001$

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- Slitless **Near Infrared Spectrometer Photometer**
- 16 HgCdTe sensors (H2RG Teledyne) operating at 90-100 K
- $\lambda_{cut} = 2.3$ $\mu$m, pixel pith $18 \times 18$ $\mu$m ($0.3 \times 0.3$ arcsec)
- 4 grisms (blue and red), 3 filters: Y, J, H
- NISP field of view $\sim 0.54$ deg$^2$
- Need $10^4 - 10^5$ spectra down to AB$= 24$ mag to calibrate the photo-z photometry

\[ \Downarrow \]

Need of a very precise redshift measurements

= the best signal fitting method and the best detectors readout mode
• Passive cooling with liquid nitrogen < 1 K/min.
• Operating temperature $T = 90$ K with 3 mK stability over all the exposures.
• Detector array can be read by 32 video outputs in parallel in full frame mode.
• Window mode → read by a single video output.
• $\sigma_R \approx 15$ e$^-$ r.m.s, $f_e \approx 2$ e$^-$/ADU
Nondestructive sampling principle

- **UTR** → cosmic rays can potentially be rejected with minimal data loss
  → high readout noise, computational power and time consuming
- **MACC** → lower RO noise, processing power and time reduction

**Euclid-NISP scientific modes:**
1) photometry MACC(3,16,4) or MACC(4,16,0) (baseline)
2) spectrometry MACC(15,16,13)

**signal fit**

**differential fit**

**optimal MACC(G,F,D)**

**optimal fit Diffs/Signal Fit with cov matrix**
1. Assuming that the frame readout noise is independent of the time sampling
Covariance matrix for groups

**diagonal terms:**

\[
\langle (\delta G_k)^2 \rangle = \langle \left( \frac{1}{m} \sum_{i=1}^{m} \delta \rho_i^{(k)} + \delta f_0 + \sum_{j=1}^{k-1} \left( \sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m - k) \delta f_i^{(k)} \right)^2 \rangle
\]

\[
C_{kk} \equiv \langle (\delta G_k)^2 \rangle = \frac{\sigma_R^2}{m} + (k - 1)(m - 1)f + (k - 1)d + \frac{(m + 1)(2m + 1)f}{6m}
\]
Covariance matrix for groups

- off-diagonal terms:

\[
\langle \delta G_k \delta G_l \rangle = \left( \frac{1}{m} \sum_{i=1}^{m} \delta \rho_i^{(k)} + \delta f_0 + \sum_{j=1}^{k-1} \left( \sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m - k) \delta f_i^{(k)} \right)

\left( \frac{1}{m} \sum_{i=1}^{m} \delta \rho_i^{(l)} + \delta f_0 + \sum_{j=1}^{l-1} \left( \sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m - l) \delta f_i^{(l)} \right)
\]

\[C_{kl} \equiv \langle \delta G_k \delta G_l \rangle = (k - 1)(m - 1)f + (k - 1)d + \frac{1}{2} (m + 1) f\]
Covariance matrix - group differences

- diagonal terms:

\[
\langle \left[ \delta (G_2 - G_1) \right]^2 \rangle = \langle \left[ d^{(1)} + \frac{1}{m} \sum_{i=1}^{m-1} (i f^{(1)}_i + (m - i) f^{(2)}_i) + \frac{1}{m} \sum_{i=1}^{m} (\rho^{(2)}_i - \rho^{(1)}_i) \right]^2 \rangle
\]

\[
D_{kk} \equiv \langle (\delta (G_k - G_{k-1}))^2 \rangle = \frac{2\sigma_R}{m} + d + \frac{(m - 1)(2m - 1)}{3m} f
\]
terms where \( l = k \pm 1 \):

\[
\langle \delta (G_2 - G_1) \delta (G_3 - G_2) \rangle = \\
\langle \left( \delta d^{(1)} + \frac{1}{m} \sum_{i=1}^{m-1} (i \delta f_i^{(1)} + (m - i) \delta f_i^{(2)}) + \frac{1}{m} \sum_{i=1}^{m} (\delta \rho_i^{(2)} - \delta \rho_i^{(1)}) \right) \\
\left( \delta d^{(2)} + \frac{1}{m} \sum_{i=1}^{m-1} (i \delta f_i^{(2)} + (m - i) \delta f_i^{(3)}) + \frac{1}{m} \sum_{i=1}^{m} (\delta \rho_i^{(3)} - \delta \rho_i^{(2)}) \right) \rangle
\]

\[
D_{k,k+1} \equiv \langle \delta (G_{k+1} - G_k) \delta (G_{k+2} - G_{k+1}) \rangle = \frac{(m^2 - 1)}{6m} f - \frac{\sigma_R^2}{m}
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**Covariance matrix**

**MACC(4,16,0)**

\[ t_{exp} = 100 \text{ sec} \quad \text{flux} = 2.5 \, e^-/\text{sec} \quad \text{frame readout noise} \, \sigma_R = 10 \, e^- \]

\[
C_{kl}^\text{simu} = \begin{pmatrix}
28.5 & 32.0 & 32.4 & 32.3 \\
32.0 & 86.9 & 91.0 & 89.9 \\
32.4 & 91.0 & 146.3 & 148.8 \\
32.3 & 89.9 & 148.8 & 204.6
\end{pmatrix} \quad C_{kl} = \begin{pmatrix}
28.2 & 31.9 & 31.9 & 31.9 \\
31.9 & 88.2 & 91.9 & 91.9 \\
31.9 & 91.9 & 148.2 & 151.9 \\
31.9 & 91.9 & 151.89 & 208.2
\end{pmatrix}.
\]

\[
D_{kl}^\text{simu} = \begin{pmatrix}
43.9 & 1.5 & 0.1 \\
1.5 & 43.6 & 2.9 \\
0.1 & 2.9 & 45.8
\end{pmatrix} \quad D_{kl} = \begin{pmatrix}
44.6 & 1.7 & 0 \\
1.7 & 44.6 & 1.7 \\
0 & 1.7 & 44.6
\end{pmatrix}.
\]

**GROUPS:**
- poisson and read noise correlates frames within groups
- poisson noise correlates groups
- \( y \)-intercept defined
- matrix \((n_g \times n_g)\), all terms \( \neq 0 \)

**GROUP DIFFERENCES:**
- poisson and read noise correlates frames within groups
- read noise correlates groups
- \( y \)-intercept NOT defined
- matrix \((n_g - 1 \times n_g - 1)\) \( l > k + 1 \) terms = 0
Optimal fit - with covariance matrix

**PHOTO**
MACC(4,16,0) $t_{exp} \approx 100$ sec

**SPECTRO**
MACC(15,16,9) $t_{exp} \approx 550$ sec

**Signal to noise ratio**

**y-intercept noise**

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Optimal RO mode - 3 signal regimes

Signal to noise ratio for spectroscopic exposures $t_{exp} \approx 550$ sec

Low signal regime
$f/\sigma_R = 0.001$

$t_{exp} = 549.00$ sec, $f = 0.01$ e$/sec$, $\sigma_R = 10.0$ e$^-$

Medium signal regime
$f/\sigma_R = 0.1$

$t_{exp} = 549.00$ sec, $f = 1.00$ e$/sec$, $\sigma_R = 10.0$ e$^-$

High signal regime
$f/\sigma_R = 10$

$t_{exp} = 549.00$ sec, $f = 100.00$ e$/sec$, $\sigma_R = 10.0$ e$^-$
2.
Temporal correlations in readout noise
Most of the publications assume that the readout noise $\sigma_R$ is independent from frame to frame and ... 

... that it is independent on the time sampling sequence.

Temporal correlations in noise induced by $1/f$ effects in the detector substrate and in the readout electronics.

If the readout noise were uncorrelated then averaging $n$ samples $\sigma^2(n)$ would scale as $\frac{\sigma^2(1)}{n}$

**GOAL:** Provide a noise model for different time samplings in nondestructive reads.

\[
\sigma^2(n) \neq \frac{\sigma^2(1)}{n}
\]
Data preprocessing

1. Window mode $8 \times 64$ pixels $\Rightarrow$ constraint on high frequency response.
2. 100 kHz pixel clock + overheads $\Rightarrow \delta = 7.12$ msec - window read time.
3. 1 exposure time $\approx 25$ min sampled up the ramp.
4. Dark current + stray light mean $= 0.8$ e$^-$/sec.
5. $\sigma_{CDS} \approx 24$ e$^-$ r.m.s. (bad grounding configuration?)
   (in full frame mode: $\sigma_{CDS} \approx 15$ e$^-$ r.m.s.)
6. Common modes subtraction:
   (in window mode we have no access to reference pixels)
   - Operation in single ended mode,
     reference channel data subtracted $\rightarrow$ noise lowered by a factor of 2.
   - Frame mean subtraction from each pixel $\rightarrow$ noise lowered by 30%.
Data processing

- Construct a Multi-accum mode:
  - \( N \) groups of \( n \) coadded frames
  - \( \Delta = \text{const}, \delta = \text{const} \)
- Coadded group of frames:
  \[
  G_k = \frac{1}{n} \sum_{i=0}^{n-1} s(t_0 + k\Delta + i\delta)
  \]
- Difference of two consecutive groups \( D_k(n, \delta, \Delta) = G_k - G_{k-1} \):
  \[
  D_k(n, \delta, \Delta) = \frac{1}{n} \sum_{i=0}^{n-1} [s(t_0 + k\Delta + i\delta) - s(t_0 + (k-1)\Delta + i\delta)]
  \]
- Assuming stationary process we measure the noise of \( D_k \) as:
  \[
  \sigma^2_F(n, \delta, \Delta) = \frac{1}{N-1} \sum_{k=1}^{N} (D_k - \langle D_k \rangle)^2
  \]
The noise does not scale as $1/n$

down

we expect some contributions from temporal correlations
Using the Wiener-Khinchin theorem

\[ \langle s(t)s(t+\tau) \rangle = \int_0^\infty \cos \omega \tau |f(\omega)|^2 d\omega \]

one can "easily" derive (Smadja et al. 2010)

\[ \sigma_F^2(n, \delta, \Delta) = \int_0^\infty (1 - \cos(\omega \Delta)) |f(\omega)|^2 \times \left[ \frac{2n}{n} + \frac{4 \cos(n\omega\delta/2) \sin((n-1)\omega\delta/2)}{\sin(\omega\delta/2)} + \frac{1 - \cos(n\omega\delta) - 2 \sin((2n-1)\omega\delta/2) \sin(\omega\delta/2)}{n^2 \sin^2(\omega\delta/2)} \right] \]

We assume that the top hat \((\omega_{min}-\omega_{max})\) filter does not affect substantially the Wiener-Khinchin theorem result.

\[
\begin{align*}
\omega_{min} &= 0.0005 \text{ rad sec}^{-1} \\
\nu_{min} &= 80 \text{ kHz} \\
\omega_{max} &= 90000 \text{ rad sec}^{-1} \\
\nu_{max} &= 15 \text{ kHz} \\
\end{align*}
\]

- \(t_{exp} = 1420\) sec
- \(\nu_{exp} \approx 0.7\) mHz
- \(\nu_{min} = \frac{1}{8 \times t_{exp}} \approx 80\) \(\mu\)Hz
- \(\nu_{max} = \frac{100}{\delta} \approx 15\) kHz
- \(\nu_{frame} \approx 7\) kHz
- \(\nu_{pix} \approx 100\) kHz
Fit results

- McWorther (1955, 1957) - surface states number fluctuations
- Voss & Clarke (1976) - resistance fluctuations induced by temperature fluctuations (metals)
- Hooge (1972) - free charge mobility fluctuations

$$|f(\omega)|^2 = A + \frac{B}{\omega^\alpha}$$

<table>
<thead>
<tr>
<th></th>
<th>$\delta$</th>
<th>$\Delta$</th>
<th>$A$</th>
<th>$B$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>set 1</td>
<td>7.12</td>
<td>0.46</td>
<td>0.388</td>
<td>119.04</td>
<td>1.14</td>
</tr>
<tr>
<td>set 2</td>
<td>7.12</td>
<td>3.56</td>
<td>0.396</td>
<td>132.84</td>
<td>1.15</td>
</tr>
<tr>
<td>set 3</td>
<td>7.12</td>
<td>14.24</td>
<td>0.412</td>
<td>139.12</td>
<td>1.34</td>
</tr>
</tbody>
</table>

- We observe a strong deviation from $1/n$ scaling law.
- For samples with higher $\Delta$, $\sigma_F^2(n)$ decreases slowly with $n \Rightarrow$ small variations of $\alpha \Leftarrow$ attributed to the imperfections of the model $|f(\omega)|^2$. 

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Noise predictions for different time samplings

The data \((n = 16)\) are perfectly reproduced by fit results.

conversion factor: \(f_e \approx 2 \text{ e}^{-/\text{ADU}}\)

<table>
<thead>
<tr>
<th>(\delta) [sec]</th>
<th>(\Delta) [sec]</th>
<th>(\sigma^2_{\text{P}}(n, \delta, \Delta)) [ADU(^2)]</th>
<th>predicted</th>
<th>measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00712</td>
<td>0.57</td>
<td>10.28</td>
<td>10.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.40</td>
<td>12.36</td>
<td>12.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.70</td>
<td>13.38</td>
<td>13.81</td>
<td></td>
</tr>
<tr>
<td>0.00712</td>
<td>21.50</td>
<td>16.42</td>
<td>15.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35.71</td>
<td>18.52</td>
<td>21.96</td>
<td></td>
</tr>
<tr>
<td>0.00712</td>
<td>57.07</td>
<td>26.10</td>
<td>26.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>71.31</td>
<td>28.47</td>
<td>29.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>106.91</td>
<td>33.82</td>
<td>37.98</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>20</td>
<td>16.56</td>
<td>16.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>15.18</td>
<td>14.14</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>24</td>
<td>12.39</td>
<td>13.52</td>
<td>(\alpha = 1.5)</td>
</tr>
<tr>
<td></td>
<td>37</td>
<td>15.15</td>
<td>14.53</td>
<td>(\alpha = 1.5)</td>
</tr>
</tbody>
</table>

set 1,2

set 3

set 3

\(\alpha = 1.5\)

\(\alpha = 1.5\)

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Contributions from thermal noise, $1/f$ noise and shot noise

\[ \sigma_f(n, \delta, \Delta) \]

- A contribution \( \Delta = 160 \text{ sec} \)
- B contribution
  - \( n = 16 \)
  - \( n = 64 \)

\[ \delta \uparrow \sim 1/\sqrt{n} \quad \downarrow \quad (?) \]
\[ \delta \uparrow \sim \text{const} \quad \downarrow \quad \text{less correlations between frames within a group} \]
\[ \Delta \uparrow \sim \text{const} \quad \uparrow \quad \text{low frequency contributions} \]

\[ \Delta_1 = 160 \text{ sec} \quad \Delta_2 = 320 \text{ sec} \]
Relative contributions from thermal noise, $1/f$ noise and shot noise

Euclid-NISP science exposure times: 
**spectroscopic** ($t_{exp} \approx 560$ sec) and **photometric** ($t_{exp} \approx 100$ sec).

<table>
<thead>
<tr>
<th>$t_{exp}$ (sec)</th>
<th>$n$</th>
<th>$\Delta$ (sec)</th>
<th>$\delta$ (sec)</th>
<th>thermal (A)</th>
<th>$1/f$ (B)</th>
<th>current</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>16</td>
<td>25</td>
<td>1</td>
<td>0.83</td>
<td>0.17</td>
<td>0.0008</td>
</tr>
<tr>
<td>100</td>
<td>16</td>
<td>68</td>
<td>1</td>
<td>0.73</td>
<td>0.27</td>
<td>0.0027</td>
</tr>
<tr>
<td>560</td>
<td>16</td>
<td>544</td>
<td>1</td>
<td>0.59</td>
<td>0.39</td>
<td>0.0154</td>
</tr>
<tr>
<td>560</td>
<td>280</td>
<td>280</td>
<td>1</td>
<td>0.22</td>
<td>0.75</td>
<td>0.0341</td>
</tr>
</tbody>
</table>

- For long spectroscopic exposures the $1/f$ noise is likely to dominate.
- It is challenging to evaluate the impact of the $1/f$ contributions on the measurement error on the fluence. A full covariance matrix must be introduced. We can guess that the $1/f$ component will at least equal and probably dominate over the thermal contribution.
### Summary

<table>
<thead>
<tr>
<th></th>
<th>time/power cons</th>
<th>y-intercept noise</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSF</td>
<td>no</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>COV</td>
<td>yes</td>
<td>low</td>
<td>high</td>
</tr>
<tr>
<td>LSFd</td>
<td>no</td>
<td>no</td>
<td>low</td>
</tr>
<tr>
<td>COVd</td>
<td>medium</td>
<td>no</td>
<td>high</td>
</tr>
</tbody>
</table>

- **Best RO mode with COV fit UTR** = uses all the frames in the ramp MACC(ng, 2, 0) assuming **time independent** readout noise.
- **$1/f$ noise** introduces **time correlations** in the frame to frame readout noise and **may dominate** in long spectroscopic exposures.
- Model of noise for a given readout mode is provided.
The conversion gain that converts digital units to electrons is determined using the Photon Transfer Curve (PTC) method.

\[ \sigma_{ADU}^2 = 2\sigma_{R, ADU}^2 + \langle S_{ADU} \rangle / f_e \]

Temporal sampling has the advantage of being independent on the pixel to pixel variations (of readout noise or quantum efficiency) within the array but places strict stability requirements on the array over a large number of exposures.

For a precision of 5% at least 1500 samples are required if the conversion gain is to be measured independently for each pixel. The pixel readout noise and conversion gain are determined with 2% and 5% accuracy respectively for each pixel.
Nonlinearity correction

Assumptions

- **Bias level constant** during the integration and readouts. Any other systematic effects (such as image persistence, bias drift, or thermal instability of the array) corrected for before the nonlinearity corrections are applied.
- Nonlinear response of HgCdTe pixels can be approximated extremely well over the range 0 to 70000 e\(^{-}\) by a quadratic function

\[ O(t) = a_0 + \varphi t - a_2 \varphi^2 t^2 \]

- Linear term coefficient \( a_1 \equiv 1 \) for all exposures. This implies that \( a_0 \) and \( a_2 \) are common to all exposures and independent of the number and absolute value of illuminations. This assumption should be revised in presence of a flux dependent nonlinearity.

Measurements

- We take several exposures up the ramp at \( M \) increasing fluences in order to cover all the dynamic range of the detector in a short acquisition time.
- \( M + 2 \) parameters \( \{\varphi_m, m=1..M\}, a_0, a_2 \) fitted by minimizing a \( \chi^2 \):

\[ \chi^2 = \sum_{i,m} \left[ \frac{O_m(t_i) - (a_0 + \varphi_m t_i - a_2 \varphi^2_m t_i^2)}{\sigma_R^2 + \sigma_P^2(m, t_i)} \right]^2 \]

- The correction applies to the absolute value of the recorded signal \( O(t) \).
- Gaussian quadrature of order $n$ accurate for all polynomials up to degree $2n - 1$ (48 nodes).

- Current contribution - Poisson noise correlations from coadding:
  
  $DI$ - signal between two groups, $di$ is the frame to frame integrated flux in electrons (see Smadja 2010 or Rauscher 2009 for derivation)

  $$\sigma^2_F \rightarrow \sigma^2_F + \langle P^2_n \rangle = \sigma^2_F + DI + \frac{(n-1)(2n-1)}{3n} di$$