

IMPACT OF THE READOUT MODES, SIGNAL FITTING AND TIME SAMPLING ON THE TOTAL NOISE OF A H2RG NEAR-IR DETECTORS

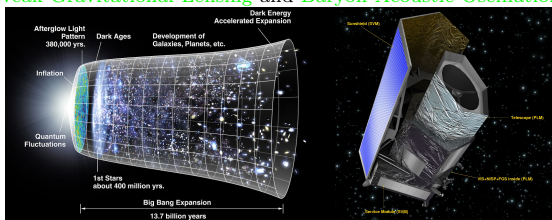
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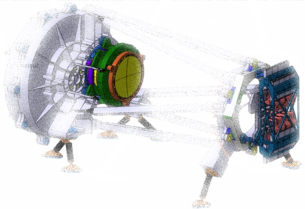
NDIP Tours
30 June 2014

Understand the **Geometry of the Universe** with 2 cosmological probes:
Weak Gravitational Lensing and **Baryon Acoustic Oscillations**



- **Dark Matter** - Λ CDM paradigm for structure formation, sum of the ν masses to a precision better than 0.04 eV when combined with Planck.
- **Dark Energy** - expansion history and structure growth (DE equation of state parameters measured to a precision of 2%)
- **Gravity** - GR vs modified-gravity theories (growth rate exponent γ with a precision of 2%)
- Wide survey: $> 15,000 \text{ deg}^2$ (36% of the total sky)
- Deep survey: $> 40 \text{ deg}^2$ (2 mag deeper than wide survey)
- **Weak Lensing**: shapes and shear of $> 30 \text{ galaxies/arcmin}^2$ for $0 < z < \sim 2$, accuracy $dz/z \sim 0.04 \rightarrow$ very high image quality and stability (ellipticity, FWHM, R2)
systematic $\sigma_{sys} < 10^{-7}$
- **Galaxy clustering**; redshifts for $> 3500 \text{ galaxies/deg}^2$ in the range $0.7 < z < 2.05$ with accuracy $dz/z < 0.001$

- Slitless Near Infrared Spectrometer Photometer
- 16 HgCdTe sensors (H2RG Teledyne) operating at 90-100 K
- $\lambda_{cut} = 2.3 \mu\text{m}$, pixel pitch $18 \times 18 \mu\text{m}$ ($0.3 \times 0.3 \text{ arcsec}$)
- 4 grisms (blue and red), 3 filters: Y, J, H
- NISP field of view $\sim 0.54 \text{ deg}^2$
- Need $10^4 - 10^5$ spectra down to AB= 24 mag to calibrate the photo-z photometry

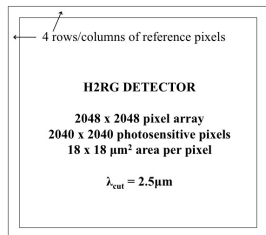
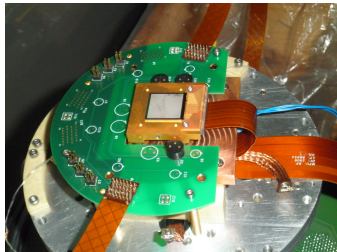
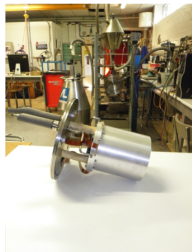


Need of a very precise redshift measurements



the best signal fitting method and the best detectors readout mode

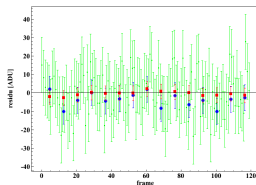
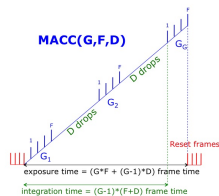
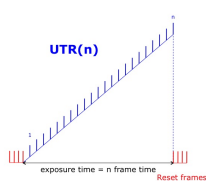
- Passive cooling with liquid nitrogen < 1 K/min.
- Operating temperature $T = 90$ K with 3 mK stability over all the exposures.
- Detector array can be read by 32 video outputs in parallel in full frame mode.
- Window mode \rightarrow read by a single video output.
- $\sigma_R \approx 15 e^-$ r.m.s, $f_e \approx 2 e^-/\text{ADU}$



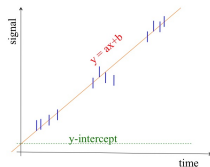
Nondestructive sampling principle

- UTR → cosmic rays can potentially be rejected with minimal data loss
→ high readout noise, computational power and time consuming
- MACC → lower RO noise, processing power and time reduction

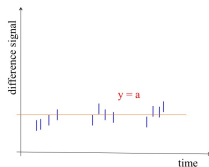
Euclid-NISP scientific modes: 1) photometry MACC(3,16,4) or MACC(4,16,0)
(baseline) 2) spectrometry MACC(15,16,13)



signal fit



differential fit



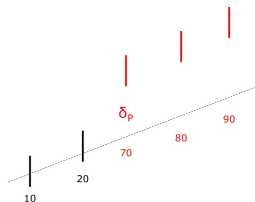
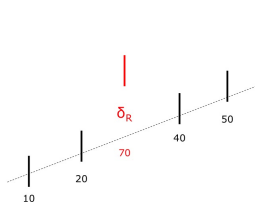
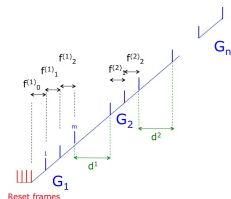
optimal MACC(G,F,D)

optimal fit Diffs/Signal
Fit with cov matrix

1.

Assuming that the frame readout noise is independent of the time sampling

Covariance matrix for groups

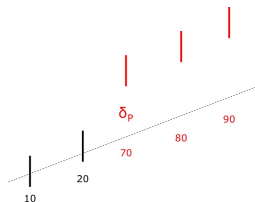
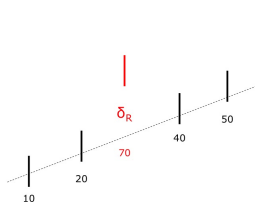
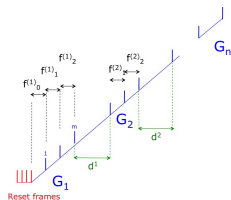


- diagonal terms:

$$\langle (\delta G_k)^2 \rangle = \left\langle \left(\frac{1}{m} \sum_{i=1}^m \delta \rho_i^{(k)} + \delta f_0 + \sum_{j=1}^{k-1} \left(\sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m-k) \delta f_i^{(k)} \right)^2 \right\rangle$$

$$C_{kk} \equiv \langle (\delta G_k)^2 \rangle = \frac{\sigma_R^2}{m} + (k-1)(m-1)f + (k-1)d + \frac{(m+1)(2m+1)f}{6m}$$

Covariance matrix for groups

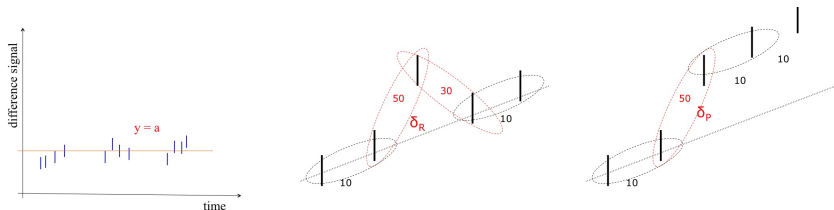


- off-diagonal terms:

$$\langle \delta G_k \delta G_l \rangle = \left\langle \left(\frac{1}{m} \sum_{i=1}^m \delta \rho_i^{(k)} + \delta f_0 + \sum_{j=1}^{k-1} \left(\sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m-k) \delta f_i^{(k)} \right) \right. \\ \left. \left(\frac{1}{m} \sum_{i=1}^m \delta \rho_i^{(l)} + \delta f_0 + \sum_{j=1}^{l-1} \left(\sum_{i=1}^{m-1} \delta f_i^{(j)} + \delta d^{(j)} \right) + \frac{1}{m} \sum_{i=1}^{m-1} (m-l) \delta f_i^{(l)} \right) \right\rangle$$

$$C_{kl} \equiv \langle \delta G_k \delta G_l \rangle = (k-1)(m-1)f + (k-1)d + \frac{1}{2}(m+1)f$$

Covariance matrix - group differences

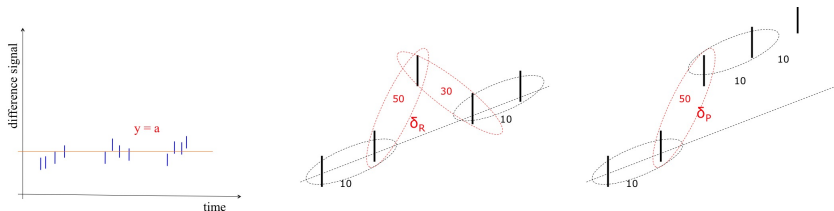


- diagonal terms:

$$\langle [\delta(G_2 - G_1)]^2 \rangle = \left\langle \left[d^{(1)} + \frac{1}{m} \sum_{i=1}^{m-1} (i f_i^{(1)} + (m-i) f_i^{(2)}) + \frac{1}{m} \sum_{i=1}^m (\rho_i^{(2)} - \rho_i^{(1)}) \right]^2 \right\rangle$$

$$D_{kk} \equiv \langle (\delta(G_k - G_{k-1}))^2 \rangle = \frac{2\sigma_R}{m} + d + \frac{(m-1)(2m-1)}{3m} f$$

Covariance matrix - group differences



- terms where $l = k \pm 1$:

$$\langle \delta(G_2 - G_1)\delta(G_3 - G_2) \rangle =$$

$$\left\langle \left(\delta d^{(1)} + \frac{1}{m} \sum_{i=1}^{m-1} (i\delta f_i^{(1)} + (m-i)\delta f_i^{(2)}) + \frac{1}{m} \sum_{i=1}^m (\delta\rho_i^{(2)} - \delta\rho_i^{(1)}) \right) \right.$$

$$\left. \left(\delta d^{(2)} + \frac{1}{m} \sum_{i=1}^{m-1} (i\delta f_i^{(2)} + (m-i)\delta f_i^{(3)}) + \frac{1}{m} \sum_{i=1}^m (\delta\rho_i^{(3)} - \delta\rho_i^{(2)}) \right) \right\rangle$$

$$D_{k,k+1} \equiv \langle \delta(G_{k+1} - G_k)\delta(G_{k+2} - G_{k+1}) \rangle = \frac{(m^2 - 1)}{6m} f - \frac{\sigma_R^2}{m}$$

MACC(4,16,0)

$t_{exp} = 100 \text{ sec}$ flux = $2.5 \text{ e}^-/\text{sec}$ frame readout noise $\sigma_R = 10 \text{ e}^-$

$$C_{kl}^{\text{simu}} = \begin{pmatrix} 28.5 & 32.0 & 32.4 & 32.3 \\ 32.0 & 86.9 & 91.0 & 89.9 \\ 32.4 & 91.0 & 146.3 & 148.8 \\ 32.3 & 89.9 & 148.8 & 204.6 \end{pmatrix} \quad C_{kl} = \begin{pmatrix} 28.2 & 31.9 & 31.9 & 31.9 \\ 31.9 & 88.2 & 91.9 & 91.9 \\ 31.9 & 91.9 & 148.2 & 151.9 \\ 31.9 & 91.9 & 151.89 & 208.2 \end{pmatrix}$$

$$D_{kl}^{\text{simu}} = \begin{pmatrix} 43.9 & 1.5 & 0.1 \\ 1.5 & 43.6 & 2.9 \\ 0.1 & 2.9 & 45.8 \end{pmatrix} \quad D_{kl} = \begin{pmatrix} 44.6 & 1.7 & 0 \\ 1.7 & 44.6 & 1.7 \\ 0 & 1.7 & 44.6 \end{pmatrix}$$

GROUPS:

- poisson and read noise correlates frames within groups
- poisson noise correlates groups
- y -intercept **defined**
- matrix $(n_g \times n_g)$, all terms $\neq 0$

GROUP DIFFERENCES:

- poisson and read noise correlates frames within groups
- read noise correlates groups
- y -intercept **NOT defined**
- matrix $(n_g - 1 \times n_g - 1)$ $l > k + 1$ terms = 0

Optimal fit - with covariance matrix

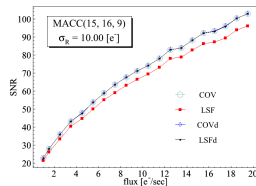
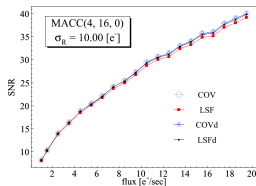
PHOTO

MACC(4,16,0) $t_{exp} \approx 100$ sec

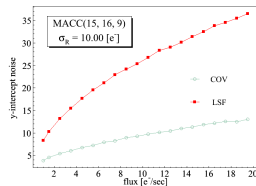
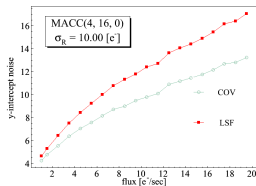
SPECTRO

MACC(15,16,9) $t_{exp} \approx 550$ sec

Signal to noise ratio



y-intercept noise



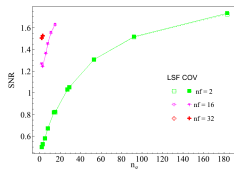
Optimal RO mode - 3 signal regimes

Signal to noise ratio for spectroscopic exposures $t_{exp} \approx 550$ sec

Low signal regime

$$f/\sigma_R = 0.001$$

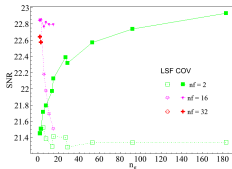
$t_{exp} = 549.00$ sec, $f = 0.01$ e⁻/sec, $\sigma_R = 10.0$ e⁻



Medium signal regime

$$f/\sigma_R = 0.1$$

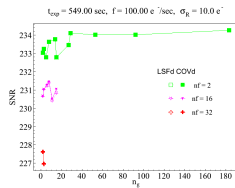
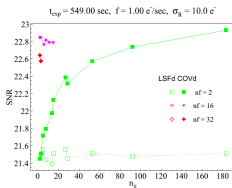
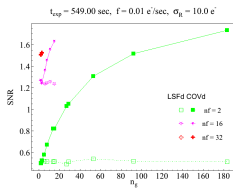
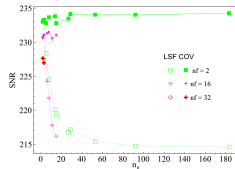
$t_{exp} = 549.00$ sec, $f = 1.00$ e⁻/sec, $\sigma_R = 10.0$ e⁻



High signal regime

$$f/\sigma_R = 10$$

$t_{exp} = 549.00$ sec, $f = 100.00$ e⁻/sec, $\sigma_R = 10.0$ e⁻



2.

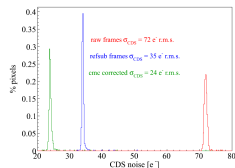
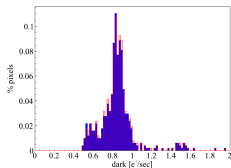
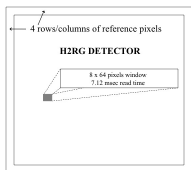
Temporal correlations in readout noise

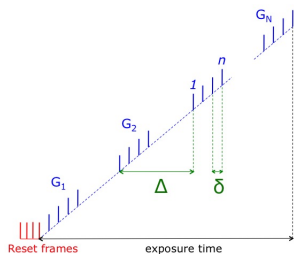
- Most of the publications assume that the readout noise σ_R is independent from frame to frame and ...
- ... that it is independent on the time sampling sequence.
- Temporal correlations in noise induced by $1/f$ effects in the detector substrate and in the readout electronics.
- If the readout noise were uncorrelated then averaging n samples $\sigma^2(n)$ would scale as $\sigma^2(1)/n$

GOAL: Provide a noise model for different time samplings in nondestructive reads.

$$\sigma^2(n) \neq \frac{\sigma^2(1)}{n}$$

- 1 Window mode 8×64 pixels \Rightarrow constraint on high frequency response.
- 2 100 kHz pixel clock + overheads $\Rightarrow \delta = 7.12$ msec - window read time.
- 3 1 exposure time ≈ 25 min sampled up the ramp.
- 4 Dark current + stray light mean = $0.8 e^-/\text{sec}$.
- 5 $\sigma_{CDS} \approx 24 e^-$ r.m.s. (bad grounding configuration?)
(in full frame mode: $\sigma_{CDS} \approx 15 e^-$ r.m.s.)
- 6 Common modes subtraction:
(in window mode we have no access to reference pixels)
 - 1 Operation in single ended mode,
reference channel data subtracted \rightarrow noise lowered by a factor of 2.
 - 2 Frame mean subtraction from each pixel \rightarrow noise lowered by 30%.





- Construct a Multi-accum mode:
\$N\$ groups of \$n\$ coadded frames
- \$\Delta = \text{const}, \delta = \text{const}\$
- Coadded group of frames:

$$G_k = \frac{1}{n} \sum_{i=0}^{n-1} s(t_0 + k\Delta + i\delta)$$

- Difference of two consecutive groups $D_k(n, \delta, \Delta) = G_k - G_{k-1}$:

$$D_k(n, \delta, \Delta) = \frac{1}{n} \sum_{i=0}^{n-1} [s(t_0 + k\Delta + i\delta) - s(t_0 + (k-1)\Delta + i\delta)]$$

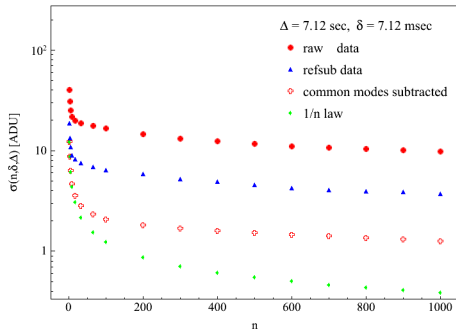
- Assuming stationary process we measure the noise of D_k as:

$$\sigma_F^2(n, \delta, \Delta) = \frac{1}{N-1} \sum_{k=1}^N \langle D_k - \langle D_k \rangle \rangle^2$$

The noise does not scale as $1/n$



we expect some contributions from temporal correlations



Noise model

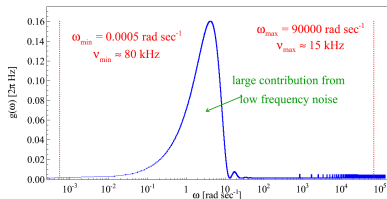
- Using the Wiener-Khinchin theorem

$$\langle s(t)s(t + \tau) \rangle = \int_0^\infty \cos \omega \tau |f(\omega)|^2 d\omega$$

- one can "easily" derive (Smadja et al. 2010)

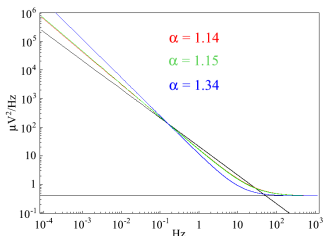
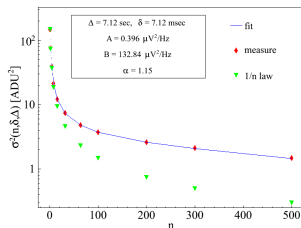
$$\sigma_F^2(n, \delta, \Delta) = \int_0^\infty (1 - \cos(\omega \Delta)) |f(\omega)|^2 \times \left[\frac{2}{n} + \frac{4}{n} \frac{\cos(n\omega\delta/2) \sin((n-1)\omega\delta/2)}{\sin(\omega\delta/2)} + \frac{1 - \cos(n\omega\delta) - 2 \sin((2n-1)\omega\delta/2) \sin(\omega\delta/2)}{n^2 \sin^2(\omega\delta/2)} \right]$$

- We assume that the **top hat** ($\omega_{min}-\omega_{max}$) filter does not affect substantially the Wiener-Khinchin theorem result.



$t_{exp} = 1420 \text{ sec}$	$\nu_{exp} \approx 0.7 \text{ mHz}$
$\nu_{min} = \frac{1}{8 * t_{exp}} \approx 80 \mu\text{Hz}$	$\nu_{frame} \approx 7 \text{ kHz}$
$\nu_{max} = \frac{100}{\delta} \approx 15 \text{ kHz}$	$\nu_{pix} \approx 100 \text{ kHz}$

Fit results



- McWorther (1955, 1957) - surface states number fluctuations
- Voss & Clarke (1976) - resistance fluctuations induced by temperature fluctuations (metals)
- Hooge (1972) - free charge mobility fluctuations

$$|f(\omega)|^2 = A + \frac{B}{\omega^\alpha}$$

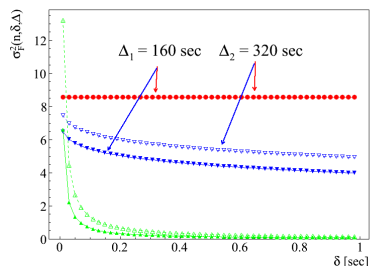
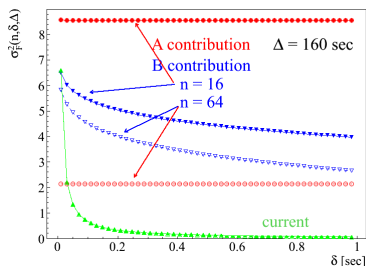
	δ [msec]	Δ [sec]	A [$\mu\text{V}^2/\text{Hz}$]	B [$\mu\text{V}^2/\text{Hz}$]	α
set 1	7.12	0.46	0.388	119.04	1.14
set 2	7.12	3.56	0.396	132.84	1.15
set 3	7.12	14.24	0.412	139.12	1.34

- We observe a **strong deviation from $1/n$ scaling law**.
- For samples with higher Δ , $\sigma_F^2(n)$ decreases slowly with $n \Rightarrow$ **small variations of α** \Leftarrow attributed to the **imperfections of the model $|f(\omega)|^2$** .

The data ($n = 16$) are perfectly reproduced by fit results.
 conversion factor: $f_e \approx 2 \text{ e}^-/\text{ADU}$

δ [sec]	Δ [sec]	$\sigma_F^2(n, \delta, \Delta)$ [ADU ²]		
		predicted	measured	
0.00712	0.57	10.28	10.23	set 1,2
	7.40	12.36	12.66	
	14.70	13.38	13.81	
0.00712	21.50	16.42	15.17	set 3
	35.71	18.52	21.96	
0.00712	57.07	26.10	26.17	$\alpha = 1.5$
	71.31	28.47	29.16	
	106.91	33.82	37.98	
0.1	20	16.56	16.56	$\alpha = 1.5$
		1	15.18	
1.5	24	12.39	13.52	$\alpha = 1.5$
		1.5	37	

Contributions from thermal noise, $1/f$ noise and shot noise



	thermal (A)	$1/f$ (B)	
n ↗	$\sim 1/\sqrt{n}$	↘	(?)
δ ↗	$\sim \text{const}$	↘	less correlations between frames within a group noise of ONE group should rise
Δ ↗	$\sim \text{const}$	↗	but the effective noise of differences is lowered low frequency contributions

Euclid-NISP science exposure times:
spectroscopic ($t_{exp} \approx 560$ sec) and **photometric** ($t_{exp} \approx 100$ sec).

t_{exp} sec	n	Δ sec	δ sec	thermal (A)	$1/f$ (B)	current
55	16	25	1	0.83	0.17	0.0008
100	16	68	1	0.73	0.27	0.0027
560	16	544	1	0.59	0.39	0.0154
560	280	280	1	0.22	0.75	0.0341

- For long **spectroscopic** exposures the **$1/f$ noise is likely to dominate**.
- It is **challenging** to evaluate the impact of the $1/f$ contributions on the **measurement error on the fluence**. A **full covariance matrix must be introduced**. We can guess that the $1/f$ component will at least equal and probably dominate over the thermal contribution.

	time/power cons	y-intercept noise	SNR
LSF	no	high	low
COV	yes	low	high
LSFd	no	no	low
COVd	medium	no	high

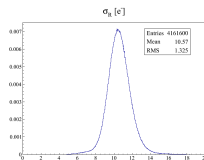
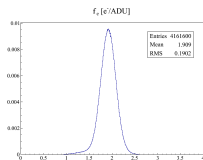
- **Best RO mode with COV fit UTR** = uses all the frames in the ramp
MACC(ng, 2, 0) assuming **time independent** readout noise
- $1/f$ noise introduces **time correlations** in the frame to frame readout noise
and **may dominate** in long spectroscopic exposures.
- Model of noise for a given readout mode is provided.

Conversion factor - Photon Transfer Curve Method

- The conversion gain that converts digital units to electrons is determined using the Photon Transfer Curve (PTC) method.

$$\sigma_{ADU}^2 = 2\sigma_{R, ADU}^2 + \langle S_{ADU} \rangle / f_e$$

- Temporal sampling has the advantage of being independent on the pixel to pixel variations (of readout noise or quantum efficiency) within the array but places strict stability requirements on the array over a large number of exposures.
- For a precision of 5% at least 1500 samples are required if the conversion gain is to be measured independently for each pixel. The pixel readout noise and conversion gain are determined with 2% and 5% accuracy respectively for each pixel.



Nonlinearity correction

1 Assumptions

- **Bias level constant** during the integration and readouts. Any other **systematic effects** (such as image persistence, bias drift, or thermal instability of the array) **corrected for** before the nonlinearity corrections are applied.
- Nonlinear response of HgCdTe pixels can be approximated extremely well over the **range 0 to 70000 e⁻** by a **quadratic function**

$$O(t) = a_0 + \varphi t - a_2 \varphi^2 t^2$$

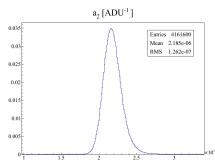
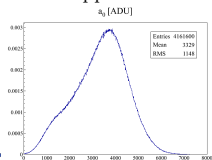
- Linear term coefficient $a_1 \equiv 1$ for all exposures. This implies that a_0 and a_2 are common to all exposures and independent of the number and absolute value of illuminations. This assumption should be **revised** in presence of a **flux dependent nonlinearity**.

2 Measurements

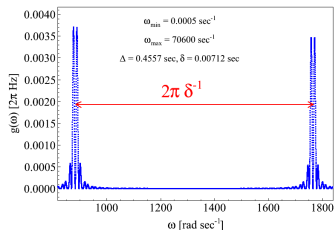
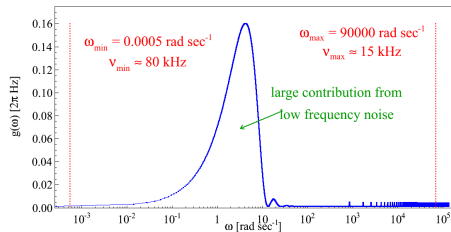
- We take several exposures up the ramp at M increasing fluences in order to cover all the dynamic range of the detector in a short acquisition time.
- $M + 2$ parameters ($\{\varphi_m, m=1..M\}$, a_0 , a_2) fitted by minimizing a χ^2 :

$$\chi^2 = \sum_{i,m} \frac{[O_m(t_i) - (a_0 + \varphi_m t_i - a_2 \varphi_m^2 t_i^2)]^2}{\sigma_R^2 + \sigma_P^2(m, t_i)}$$

- The correction applies to the absolute value of the recorded signal $O(t)$.



Fitting to Data - Gauss Legendre Method



- Gaussian quadrature of order n accurate for all polynomials up to degree $2n - 1$ (48 nodes).
- Current contribution - Poisson noise correlations from coadding:
 DI - signal between two groups, di is the frame to frame integrated flux in electrons (see Smadja 2010 or Rauscher 2009 for derivation)

$$\sigma_F^2 \rightarrow \sigma_F^2 + \langle P_n^2 \rangle = \sigma_F^2 + DI + \frac{(n-1)(2n-1)}{3n} di$$

